THÆ

ELEMENTS

OR

PRINCIPLES

OF

GEOMETRIE:

COLLECTED

From ſeveral Authors and reduc'd into a ſhorter Method.

LONDON:

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TO THE

INGENUOUS and HOPEFUL

GENTLEMAN,

Mr. RALPH FREMAN,

SON OF

RALPH FREMAN,

OF *ASPEDEN* HALL

ESQUIRE;

THESE

ELEMENTS,

WHICH WERE AT FIRST

PREPARED FOR HIS USE,

ARE NOW WITH DUE RESPECT

PUBLICLY

DEDICATED.

Advertiſement.

THE deſign of this *Treatiſe* being to reduce the *Elements* or *Principles* of *Geometrie* into a ſhort Compaſs, for the benefit of thoſe that haſten to the Practical parts of this Studie, it ſeem'd fit to lay down the following *Definitions*, for the moſt part, as they uſually are expreſt, rather than ſpend time in giving reaſons for the contrary. Though we muſt confeſs with the Ingenuous *Borellio*, that the Angle of the Tangent (or a *mixt* Angle) is indeed no Angle. And perhaps *Geometrie* it ſelf may be better defin'd, the Art of meaſuring *Space*, rather than *Bodies*: And then, a *Surface*

(being the Boundary of ſuch a Space) and Lines, the Boundaries of ſuch Surface, as Points of ſuch Lines, will all naturally appear to be immaterial things. Laſtly, the Definition of *Reaſon* and *Aliquot parts* do not extend to incommenſurable Quantities, which are omitted in this ſhort Collection. And ſome Propoſitions about Proportion, according to the deſigned brevity, are rather illuſtrated, than ſtrictly demonſtrated. Thoſe that deſire to be curious in theſe particulars can have recourſe to larger Authors. This Summary was thought ſufficient for an *Introduction*.

THE

THE
Elements *or* Principles
OF
GEOMETRIE.

DEFINITIONS.

CHAP. I.

Of Lines.

GEOMETRIE is the Art of Measuring Bodies: In order to which end, it considers three principal things that belong to a Bodie, namely, Superfice or Surface, Lines, and Points: Which in their proper notion, do signifie, the Out-side, *the* Edg, *and the* Corners *of a Bodie; and so, are real parts of it: But*

in the ſenſe that Geometrie uſes them, they do not properly belong to a Bodie, only are applied to it by our imagination. For if a Bodie be dipt in water, though the water touches it every where, yet we may fancy a diviſion between it and the water, which imaginary diviſion, is called a Surface. *And thus, every imaginary diviſion of a Bodie, begets a* Surface; *(between the parts ſo divided:) of a* Surface, *begets a* Line; *of a* Line *begets a* Point. *And ſince whatſoever divides a thing, terminates or bounds the parts ſo divided; therefore,*

1. *A* Point *is ſaid to be bound of a* Line.

2. *A* Line *the bound of a* Surface. *And,*

3. *A* Surface *the bound of a Bodie.*

The firſt, *having neither length nor breadth: The* ſecond, *length without breadth; and the* third, *length and breadth without depth.*

4. *A* Right-Line, *is that which lies ev'n between its two bounding Points; and by conſequence, marks out the ſhorteſt way from one Point to another.*

5. *A*

DEFINITIONS.

5. *A* **Plane Surface** *is that which lies ev'n, between its two bounding Lines, and becauſe it is ſtretcht out ſtrait (like the head of a Drum) therefore it is the ſhorteſt that can lie between the ſame two Lines.*

6. *An* **Angle**, *is the corner* [a] *that is made by the meeting of two Lines:* 1. *Which Angle is greater or leſs, according as the ſaid Lines* [a b, a c] *lean nearer or ſtand further off from one another, which is call'd their* Inclination: 2. *So that if ſeveral Lines* [a b, a c, *and* d e, d f] *have the ſame* Inclination, *the Angles which they make* [a = d] *are equal.*

Fig. I.

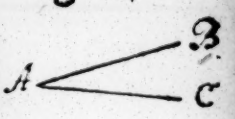
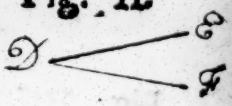
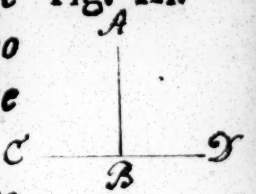


Fig. II.



7. *A* **Right-Angle**, *is when one Line* [a b] *ſo meets another* [c d], *that the two Angles on each ſide,* [a b c, *and* a b d], *are equal to each other.*

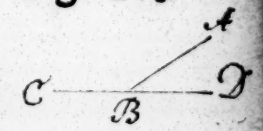
Fig. III.



Perpendicular: *In this caſe, the Line* A B, *is ſaid to be Perpendicular to* C D.

8. *An* **Obtuſe** *Angle* [a b c] *is that which is bigger than a* Right.

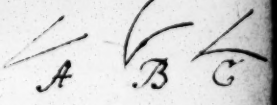
Fig. IV.



9. *An* **Acute** *Angle* [a b d] *is that which is leſs than a* Right.

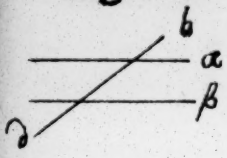
10. *A* **Right-Lin'd-Angle** [a] *between two Right-Lines, a* Curv-lin'd, [b] *a* mixt [c].

Fig. V.



11. **Parallel**

Fig. VI.



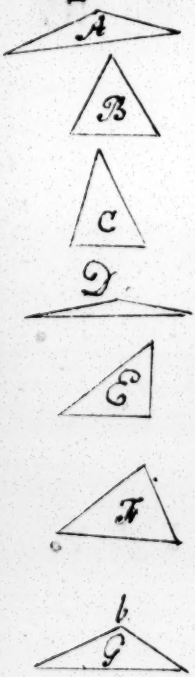
11. **Parallel** *Lines* [α β] *are those which lean not at all towards one another. So that if they were drawn out infinitely, they won'd never meet.*

CHAP. II.

Of Figures. PART I.

12. A **Plane Figure**, *is a Plan Surface inclos'd in one o more Lines.*

Fig. VII.



13. *A* **Triangle**, *is a Figure bounde with three Lines*, A.

14. **Equilateral**, *which has all sid equal*, B.

15. **Isoscele**, *or equal legg'd, which h two sides equal*, C.

16. **Scalene**, *which has no sides qual*, D.

17. **Right-Angled**, *which has o Right-angle*, [a] E.

18. **Acute-Angled**, *which has all i Angles Acute*, F.

19. **Obtuse-Angled**, *which has o Obtuse Angle* [b] G.

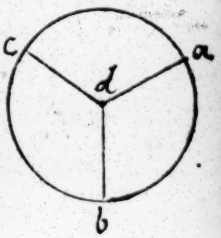
20.

DEFINITIONS.

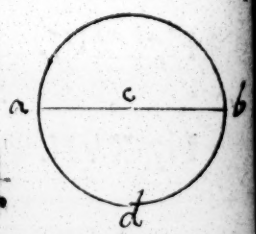
20. *A* **Parallelogram**, *is a four sided Figure, whose two opposite sides* [a b, c d *or* a c, b d] *are Parallel Lines.* Fig. VIII.

Of a Circle. PART II.

21. A **Circle** *is a plane Figure bounded with one Line, call'd the* Circumference [a b c a], *to which all the Lines* [d a, d c, d b] *that can be drawn from one Point* [d] *in the middle of the Figure, are equal to one another; these Lines are call'd* Radius's, *and the Point in the middle, the* Center. Fig. IX.

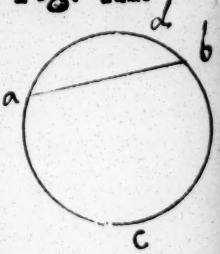


22. *The* **Diameter** *of a Circle, is a Right-Line* [a b] *passing through the Center,* [c] *bounded at each end with the Circumference, and dividing the Circle into two equal parts.* Fig. X.



23. *A* **Semicircle** *is the Figure* [a d b] *contain'd between the Diameter, and half the Circumference.*

24. *A* **Segment** *(of a Circle) is a Figure contain'd between a Right-Line (call'd a* Chord*) and any part of a Circumference,* [a c b, *or* a d b] *call'd an Arch.* Fig. XI.



25. Equal

25. **Equal Circles**, *are such, whose Diameters or Radius's (that is Semidiameters) are equal.*

Fig. XII.

26. **Circles** *are said to* Touch, *when they do only touch, and not cut one another.*

Fig. XIII.

27. *A* **Right-Line** [a b] *is said to touch a Circle, when being continu'd it does not cut the Circle: This is call'd a* Tangent.

Fig. XIV.

28. *An* **Angle** [c] *is said to* stand upon *that part of a Circumference* [a b] *which is opposit to it.*

Fig. XV.

29. *An* **Angle of the Segment** [a b d] *is made by the Circumference and a Right-Line cutting it.*

30. *An* **Angle in the Segment** [c] *is made by two Right-Lines* [a c, b c] *rising from the* Angles of the Segment, *and meeting in the Circumference.*

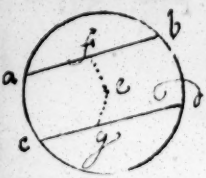
Fig. XVI.

31. *An* **Angle** *of* Contact, [b] *is between the Tangent and Circumference.* F. 13.

Fig. XVII.

32. *The* **Sector** *of a Circle,* [a b c] *is a Figure made by two Radius's and part of the Circumference.*

Fi. XVIII.

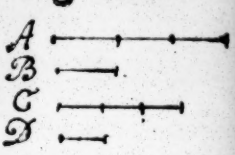


33. **Right-Lines** [a b, c d] *are said to be* Equidistant *from the Center,* [e], *when Lines* [e f, e g] *drawn Perpendicular from the Center to them, are equal.*

CHAP. III.

Of Proportion.

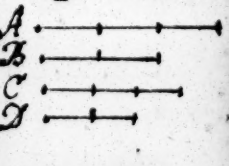
34. A **Multiplied** *Magnitude*, [a] *is that which contains another Magnitude*, [b] *a certain number of times preciſely.* Fig. XIX.



35. *An* **Aliquot** *part, or ſimple*; [b] *which being repeated a certain number of times, equals, or meaſures out another* [a] *Magnitude preciſely.*

(1.) Like Aliquot *parts*, [b, d] *are ſuch, as being equally repeated do meaſure out their reſpective wholes*, [a, c.]

(2.) Like Parts, *are thoſe that are equally contain'd in their reſpective wholes: Thus* B *and* D *are* like parts, *becauſe* B *is contain'd once and a half in* A, *and* D *once and a half in* C. Fig. XIX. Fig. XX.



36. **Ratio** *or* Reaſon, *is the compariſon of two quantities* [a, b] *one with another; whereby, one is ſaid to be bigger or leſs than the other: In which compariſon, that which precedes*, [a] *is call'd the* Antecedent, *and the other* [b] *the Conſequent.* Fig. XIX.

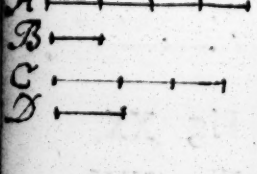
37. *Thoſe*

DEFINITIONS.

37. *Thoſe* **Quantities** *only admit of a* Reaſon, *which being Multiplied may exceed each other.*

Fig. XIX. 38. *The* **Reaſons** *(between* A, B, *and* C D) *are ſaid to be the* ſame (equal, *or* like) *when both the conſequents* (B *and* D) *are* like parts *of their reſpective Antecedents* (A *and* B). *That is, ſince no quantity can be ſaid to be big or little, but as it is compar'd to another, therefore if* B *and* D *are like parts of* A *and* C, *then* A *is ſaid to be as big, in reſpect of* B, *(or, to have the* ſame reaſon *to* B) *as* C *has in reſpect of* D: *Thus expreſt,* A . B :: C . D, *or thus,* $\frac{A}{B} = \frac{C}{D}$.

Fig. XXI. 39. *One* **Reaſon** *is ſaid to be greater or leſs than another, when one of the Conſequents* [b] *is more exceeded by its Antecedent* [a], *than the other* [d] *by its Antecedent* [c]. *That is, A is bigger in reſpect of* B, *than* C *is in reſpect of* D, *or, the* Reaſon *of* A *to* B, *is bigger than the reaſon of* C *to* D, *which is thus expreſt;* $\frac{A}{B} \sqsubset \frac{C}{D}$.



40. *The*

40. *The* **Equality** *of Reaſons, (mentioned in* def. 38.) *is call'd* Proportion: *That is,* A, B; C, D *are ſaid to be* Proportionals. *And in* def. 39. A, B; C, D *are* Unproportionals.

41. **Continual** *Proportionals, are when the middle term (or quantity) is taken twice: as when,* A *is as much bigger than* B; *as* B, *is than* C; *which is thus expreſt,* A, B, C, ∴

42. *In the foregoing caſe the reaſon of the firſt term to the third, is ſaid to be* Duplicate *to the reaſon of the firſt to the ſecond; or of the ſecond to the third, (for both theſe* reaſons *are the ſame, by* def. 38.) *and if there be more terms added,* viz. A, B, C, D, *&c.* ∴ *the Reaſon of the firſt* [a] *to the fourth* [d], *is ſaid to be* Triplicate, *&c.*

43. *But if the terms* ABC *are not Proportionals (as in* def. 39.) *then the Reaſon of* A *to* C, *is ſaid to be* compounded *of the Reaſon of* A *to* B, *and of* B *to* C, *thus expreſt,* $\frac{A}{C} = \frac{A}{B} + \frac{B}{C}$.

44. *The* **Homologous** *terms in any Proportion, are the two* Antecedents, *or the two* Conſequents: *Thus* A, C, *and* B, D,

D, (*in* Fig. 20.) *are the* Homologous *terms.*

Fig. XXII. 45. *Like* **Right-Lin'd** *Figures,* [a b c, d e f], *are such as have equal Angles, and the sides about those equal Angles, Proportional, as if the Angle* B, *be supposed to be equal to the Angle* E, *then the sides* AB . BC :: DE . EF, *and so of the other Angles and sides.*

46. **Reciprocal** *Figures, are when you compare the sides of one Figure to the sides of the other, and the Antecedents and Consequents of the Reasons, are in both Figures.*

Fig. XXIII 47. *The* **Height** *of any Figure, is a Perpendicular Line* [a b] *drawn from the top of it* [a] *to the basis* [c d].

CHAP. IV.

Of the Power of Lines.

Fig. XXIV 48. A **Rectangle** *is a Parallelogram* (def. 20.) *whose Angles are Right.*

Fig. XXV. 49. *A* **Square** *is a Rectangle that has all its sides equal, these are also call'd the Powers of Lines.*

CHAP. V.

Of Surfaces.

50. A **Right-Line**, [a b] *is* Right *(or Perpendicular) to a* Plane [c d] (def. 5.) *when all the Lines* [a c, a d, *&c.*] *that can be drawn from that Point* [a] *where it touches the Plain, upon the ſaid Plane do make Right-angles with it,* viz. *when* CAB, DAB; *&c. are Right-angles* Fi. XXVI.

51. *The* **Inclination** *of a Right-Line* [a b] *to a Plane,* [c d] *is meaſured by the Angle* B A E, *(A E being the points) where the Line that leans, and the Perpendicular, do touch the Plane.* F. XXVII.

52. *One* **Plane** [a b] *is* Right *to another Plane,* [c d] *when all the Lines in the firſt Plane* [a b] *that are Perpendicular to the common ſection,* [b e] *are alſo Perpendicular to the other Plane* [c d] (def. 50.) F. XXVIII.

53. *The* **Inclination** *of one Plane* [a b] *to another* [c d] *is meaſured by the Angle* [g] *between two Lines* [h g, f g] *in each Plane, which are Perpendicular to the common interſection* [e b]. Fig. XXIX.

54. *This* **Inclination** *will be* equal, *(or* like*) in ſeveral Planes when this Angle is equal.*

55. **Parallel** *Planes are ſuch as have no Inclination to one another.*

56. *A* **Solid** *Angle, is a Corner made by the meeting together of ſeveral Plane Angles (three at the leaſt) in one Point.*

CHAP. VI.

Of Solids, or Bodies.

57. A **Solid** *or* Body, *is that which has Length, Breadth, and Depth.*

58. Like **Solid** Figures, *are ſuch as are contained under an equal number of like plain Figures.*

59. Equal and like **Solid** Figures, *are ſuch as are contain'd under an equal number of like and equal plain Figures.*

60. *A* **Pyramid**, *is a Solid Figure, whoſe ſides are plain Triangles, their ſeveral tops meeting together in one Point*

61. *A*

61. *A* **Prism**, *is a Solid Figure, the two opposite sides (or ends)* [a b c, d e f] *of which are* like, *equal, and Parallel; and all the other sides* [a b d e, *&c.*] *are Parallelograms.* Fig. XXX.

62. *A* **Sphere** *is a Solid Figure bounded with one Surface; to which (Surface) all the strait Lines that can be drawn from one Point within the Figure, call'd the* Center, *will be equal.*

63. *The* **Axis** *of a* Sphere, *is that resting Right-Line, about which, if a Semicircle be turn'd, it will beget a Sphere.*

64. *The* **Center** *of a Sphere, is the middle of this* Axis.

65. *The* **Diameter** *of a Sphere, is a Right-Line passing through the Center, and bounded at each end in the Surface of the Sphere.*

66. *A* **Cone**, *is a Solid Figure, rising from a Circular base* [c d] *according to Right-Lines,* [c a, d a] *and ending in a Point,* [a] *which is call'd the Vertex, or Top.* Fig. XXXI.

67. *The* **Axis** *of a Cone is a Right-Line drawn from the Top,* [a] *to the Center* [b] *of the base* [c d].

68. *A* **Right Cone**. *See* def. 71.

Fi. XXXII. 69. *A* **Cylinder** *is a Solid Figure, rising from a Circular base* [c d] *(or Circle) according to Right-Lines,* [c e, d f] *and ending in an equal Circle* [e f].

70. *The* **Axis** *of a Cylinder, is a Right-Line,* [a b] *joyning together the Centers of the two Circles.*

71. *A* Right **Cone** *or* **Cylinder**, *is when the Axis is Perpendicular to the base.*

72. Like **Cones** *or* **Cylinders**, *are such, whose Axes, and the Diameters of their bases, are proportional.*

F. XXXIII. 73. *A* **Cube** *(or Dye) is a Solid Figure contain'd under six equal Squares.*

74. *A* **Tetraedrum**, *contain'd under Four Triangles, equal and equal sided.*

75. *An* **Octaedrum**, *contain'd under Eight Triangles, equal and equal sided.*

76. *A* **Dodecaedrum**, *containd under Twelve Pentagons, equal and equal sided.*

77. *An* **Icosaedrum**, *contain'd under Twenty Triangles, equal and equal sided.*

These five last only, are call'd *Regular* Bodies.

F. XXXIV. 78. **Parallelepipid** (P pp) *is a Solid Figure, contain'd under six Parallelograms, the opposite of which, are Parallel.*

79. *One*

AXIOMS.

79. *One Figure is ſaid to be* Inſcrib'd *in another, when all the Angles of the Figure inſcrib'd, touch either the Angles, Sides or Planes, of the other Figure.*

80. *A* **Figure** *is* Conſcrib'd *(or* Circumſcrib'd*) when either the Angles, Sides or Planes of the outward Figure, touch all the Angles of the Figure that is inſcrib'd.*

Note, That theſe *Definitions*, though put all together, for the convenience of references, will be beſt perus'd ſeverally, before each reſpective *Chap.* in the Book, to which they belong.

AXIOMS.

1. *BEtween two Points, there cannot lye more than one Right-Line, (nor between two Lines, more than one Right Surface) but they will be* Coincident; *that is, become one. Therefore ſuch Lines (and Surfaces) can't have one common Segment, that is, one part or portion, common to two or more of them.*

2. *All* Right Angles *are equal to one another.*

AXIOMS.

3. *Parallel Lines,* [$\alpha\beta$] (*having no Inclination to one another*) *have the ſame Inclination to a third Line* [β].

4. *Thoſe things which being laid upon one another, do meet in all parts, are equal. The Converſe of this* (*to wit, that equals being laid upon one another, will meet*) *is true in Lines and Angles, but not in Figures.*

5. *A* Whole *is greater than any* part *of it; and equal to all its* parts *taken together.*

6. *Things that are equal to a third, are equal to one another: If* A *be equal to* C, *and* B, *be eqnal to* C, *then* A *is equal to* B.

7. *The* halfs (*or* doubles) *of things that are equal, are equal, and ſo are any multiples,* &c.

8. *If you add, or take away equal parts from things that are equal, the remainders will be eqnal.*

9. *If you add, or take away equal parts from things that are* unequal, *the remainders will be* unequal.

Of

AXIOMS.

Of Proportion.

10. *Thoſe things that have the ſame* Reaſon *to a third thing, (or to things that are equal,) are themſelves equal: That is, thoſe things that are equally great in reſpect of another, are equal between themſelves; and convertedly.*

11. *And if* A, *be greater in reſpect of* C, *than* B *is in reſpect of* C, *then* A *is greater than* B. Fi. XXXV.

12. *Or (which is the ſame in other words)* A *is leſs than* B, *if the* Reaſon *of* C *to* B, *be greater* (def. 39.) *than the* Reaſon *of* C *to* A: *That is, if* A *be leſs in reſpect of* C, *than* B *is, in reſpect of the ſame* C, *then* A *is leſs than* B. F. XXXVI.

13. *Thoſe* Reaſons *that are* equal *(or* like) (def. 38.) *to a third* Reaſon *are equal between themſelves, and thoſe that are unequal to a third, are unequal to all that are equal to this third.*

14. *A* Whole, *and all its parts together, have the ſame* Reaſon *to a third thing.*

= Equal to.
⊏ Greater.
⊐ Leſs.
+ Added to.
— Subtracted, or divided by.
× Multiplied by.
:: Like.
÷ Continued Proportion.
∟ Right-Angle.
∠ Angle, or Acute-Angle.
△ Triangle.
○ Circle.
□ Square.
▭ Rect-angle.
Perp. Perpendicular.
Pll. Parallel.
Pgr. Parallelogram.
Ppp. Parallelepipedon.
Hyp. Hypotheſis, or Suppoſition.
Pre. Precedent Propoſition.
QED which was to be Demonſtrated
QEA which is absurd
QEF which was to be done
QEO which was to be shown

THEO-

THEOREMS.

[Note. *In these* Theorems *(or Speculative Principles) it is enough to suppose that Parallel Lines can be drawn, Angles fram'd, and Circles describ'd. The manner how to do them, will be shew'd and Demonstrated in the* Problems *(or Practical Principles) that come after.*]

CHAP. I.

Of Lines. PART I.

Theorem I.

One right Line [a b] *falling upon another* [c d] *makes either two* ∟s, *or such as are* = 2 ∟.

IF A B stands Berp. it makes [a] 2 ∟s ABC, ABD. If it lean, (as EB,) the Angles EBC. EBD, take up the same place as the former 2 ∟ ones had done, and by consequence [b] are equal to them. *Q. E. D.*

[a] *Def.* 7.

[b] *Ax.* 4.

II.

Theorem II.

That [c d] *is a right Line, on which another* [a b] *ſtanding, makes* 2 ∟s *or ſuch as are* = 2 ∟.

IF CD, be not a right Line, then the right Line will fall either under or over it, as CBE. Now the Angles ABC, ABD = [c] (2 ∟ = [d]) ABC, ABL, *i. e.* a part equal to the whole, *Q. E. A.* Therefore C D will be the right Line. *Q. E. D.*

[c] *Sup.*
[d] *Præ.*

Theorem III.

The oppoſite Angles of croſſing Lines (call'd Head Angles) are equal to one another, as a = b, *and* c = d.

D + A = [e] (2 ∟ = [e]) D + B, therefore (D being common) A is = B. Alſo A + C = [f] (2 ∟ = [f]) A + D, therefore (A being common) C is = to D. *Q. E. D.*

[e] *I.*
[f] *Ax.* 9.

The reaſon is this, If HF cut EG Perp. all the Angles are ∟ (by I) and therefore equal (Ax. 2.) ſo then, C = D and A = B. Now if the point O remaining fixt, H F be leaned aſide, F will come to E at the ſame time that H comes to G, therefore whatſoever is loſt out of the Angles B and A, is equally gain'd by the other

other Angles C and D, so [g] that B will always be equal to A and C = D. Q.E.O. [g] *Ax.* 8.

Theorem IV.

A right Line [γ δ] *cutting Parall.* [α β] *makes all the oppoſite Angles equal*, viz. a = b = c = d. e = f = g = h

FOr α β have [g] the ſame inclination to γ δ, and therefore [h] make = Angles with it. That is, A = C and F = H. But A [i] = B and C [k] = D, alſo E = F and G = H, and therefore [k] they are all = one another, viz. A = B = C = D, and E = F = G, = H. *Q.E.D.*

[g] *Def.* 11. [h] *Def.* 6. [i] *Pre.* [k] *Ax.* 6.

Theorem V.

And two of the oppoſite Angles (whether internal, [f c. b g] *or external)* [a h, e d] *are equal to* 2 ∟.

FOr, F. A (= [l] C) = [m] 2 ∟ Therefore
B. E (= [l] G) = [m] 2 ∟

F C = } 2 ∟, and ſo of the reſt. *Q.E.D.*
B G = }

[l] *Pre.* [m] *I.*

VI.

Theorem VI.

If the oppoſite Angles [a b c d e f g h] *be equal, the Lines* α β *are* P ll.

n *Sup.*
o *Def.* 6.
p *Ax.* 3.
q *Def.* 11.

SInce the Angle A = [n] C, the Lines α β have the ſame [o] inclination to a third Line γ δ; and therefore [p] no inclination to one another, that is [q] to ſay, are Parallels. *Q. E D.*

Theorem VII.

Alſo if the oppoſite Angles are = 2 ∟ [f c, *or* b g, *or* a h, *or* e d,] *the Lines* α β *are Parallels.*

r *Sup.*
s *I.*
t *Ax.* 8.
u *Pre.*

FOr, F C = [r] (2 ∟ = [s]) F A, therefore taking away F which is common, there remains [t] C = A. Wherefore [u] α β are Parallels. *Q. E. D.*

Theorem VIII.

Lines Parall. α β *to a third,* δ *are Parall. to one another.*

w *Ax.* 3.

IF you deny it, let α be ſuppos'd inclin'd to β, therefore [w] δ is inclin'd to β after the ſame

me manner, contrary to our ſuppoſition, hich is abſurd; therefore *a* is not inclin'd to and by conſequence is Parall. to it. *Q. E. D.*

CHAP. II.

Of Figures. PART. I.

Theorem IX.

a Triangle [a b c] *the external Angle* [f c b] *is equal to the two internal oppoſite Angles* [a b.]

Et *α β* be drawn Parallel to A B. Then the Angle D = [x] A, and E = [x] B. Therefore [y] both together, are equal to A + B. *Q. D.*

x *IV.*
y *Ax.* 5.

Theorem X.

e three Angles of every Triangle, are equal to two right Angles.

r, C D = [z] 2 ∟ and D = [a] A + B, therefore C A B = 2 ∟. *Q. E. D.*

z *I.*
a *Pre.*

XI.

Theorem XI.

Two sides [a b, b c] *of any Triangle a*
greater than the third [a c.]

[b] *Sup.*
[c] *Def.* 4.

FOr A C being [b] a right Line, marks [c] o
the shortest way between the points A a
C, and therefore is shorter than A B C, whi
lye between the same points. *Q. E D.*

Theorem XII.

In a Triangle [b c d] *equal Angles* [c
are subtended by equal sides [b c, b d.]

SUppose the whole Triangle to be turn'd
backside forward, (represented by ꝗ ↄ
and laid on the foreside B C D, so that the p
ꝗ may fall upon C, and ↄ upon D: now bec
the Angle C = [d] D, therefore [e] the Angl
shall meet exactly with C, and ↄ with D;
by consequence the Line ꝗ ꝗ shall fall upon
and ↄ ꝗ upon D B. Lastly, the point ꝗ
fall upon B, (for should it fall any wh
else, as in E, then the Lines ꝗ ꝗ and ↄ ꝗ wo
not fall upon C B and D B, against what has b
demonstrated) therefore C B = [f] D B. *Q*

[d] *Sup.*
[e] *Ax.* 4.
[f] *Ax.* 1. *and* 4.

Theorem XIII.

In a Triangle equal sides [a b, b c] *subtend equal Angles.* [a c]

BEcause the Lines B A = [g] B C, therefore both of them in the point B, are equally distant from the Angles A and C, as are also both of them in their points A and C, (for the line A C is a common measure to both their points;) since therefore A B and B C, are equally distant from the Angles A and C in both their extream Points, and are equal [h] to one another, it follows [i] that the Angles subtended by them, A and C, are also equal. *Q. E. D.*

g *Sup.*
h *Sup.*
i *Def.* 6. *Cas.* 3.

Theorem XIV.

In a Triangle the greater side [b c] *subtends the greater Angle.* [a]

IF not, let the Angle C be suppos'd = to A, then [k] A B will be = to C B against the supposition. *Q. E. A.* Suppose then C ⊏ A, and take the Angle A C D = A, therefore [k] the side C D = A D and adding D B to both, C D B will be = [l] to A D B. But C D B ⊏ [m] (C B ⊏ [n]) A D B. *Q. E. A. viz.* that the same C D B should both = to and ⊏ than A D B.

k *XII.*
l *Ax.* 8.
m *XI.*
n *Sup.*

XV.

Theorem XV.

In a Triangle the ⊏ Angle [a] *is ſubtended by the ⊏ ſide.* [bc]

IT is ſuppos'd that the Angle A is ⊏ than C, and it muſt be prov'd that the ſide BC is ⊏ BA; for firſt, if BC were = BA, then the Angle A would be = ° C, contrary to the Suppoſition. *Q. E. A.* Or 2*dly*, if BA were ⊏ BC, then the Angle C ᵖ would be ⊏ A, againſt the Suppoſition alſo. Since therefore BC is neither = nor ⊐ BA, it follows that it muſt be ⊏ BA. Q. E. D.

° *XIII.*

ᵖ *Pre.*

Theorem XVI.

In two Triangles if the ſides be =, the Angles are =.

FIrſt I prove the Angle A = D, for ſince the ſides AB, AC are = to DE, DF, and the ſubtendent B = CF, it follows ᵠ that the ſides, AB, AC, and DE, DF, have the ſame inclination and by conſequence that the Angles which they contain, A and D, are =. *Q. E. D.*

ᵠ *Def.* 6.

The ſame proof will ſerve for the other Angles.

XVI

Theorem XVII.

In two Triangles, where two ſides [b a c *and* e d f] *and one Angle* [a d] *(between thoſe ſides) are* =, *all the reſt, both ſides and Angles, are* =.

SInce the ſides B A C and E D F, are =, and alſo the Angles A and D, therefore being laid upon another, both the ſides and Angles ſhall exactly meet, [r] and by conſequence the point E ſhall fall upon B, and F upon C; therefore [f] the line E F ſhall exactly meet with B C, and by conſequence [t] be = to it: and therefore [t] all are =. *Q. E. D.*

[r] *Ax.* 4.
[f] *Ax.* 11.
[t] *Pre.*

Theorem XVIII.

In an equal legg'd Triangle (def. 15) *a line* [b d] *drawn from the top, and cutting the baſe* [a c] *in the middle* [d], *is perp. to the ſame baſe.*

FOr in the two Triangles, A B D and B D C, the ſide B A = [u] B C, and A D = [u] D C, and B D is common to both, therefore [w] the reſpective Angles are = to one another; and particularly thoſe two at D, which are therefore [x] ⌊_; and by conſequence [y] B D is perp. to A C. *Q. E. D.*

[u] *Sup.*
[w] *XVI.*
[x] *I.*
[y] *Def.* 9.

Theorem XIX.

In two Triangles, where two sides [b a c, e d f] *are* =, *and a third side* [e f,] ⊏ *the Angle* [d] *subtended by this third side, is also* ⊏.

THis follows manifestly from *def.* 6. *cas.* 3. But may be demonstrated (if need were) as XVII, by laying one upon the other.

Theorem XX.

The converse of the former. If two sides [b a c, e d f] *be* = *and one Angle* [d] ⊏, *the subtendent* [e f] *of this Angle, shall be* ⊏ *also.*

IF you deny it, let the subtendent E F be =, or ⊐ than B C, therefore the Angle D will be = or ⊐ than A, contrary to the supposition; wherefore it remains that the subtend. E F should be ⊏ than B C. *Q. E. D.*

Theorem

Theorem XXI.

In two Triangles, if one ſide [a b, α β] *and two Angles* [α, a; β, b] (*adjoyning to this ſide*) *be* =, *all the reſt ſhall be* =.

SEt αβ upon AB, and they ſhall meet [z]; alſo the Angles α and β being [a] = to the Angles A, and B, ſhall exactly meet [z] with them. Laſtly, The point ∫ ſhall fall upon C (for ſhould it fall any where elſe, as in E, then the ſide β∫ would not fall upon the ſide BC, as has been demonſtrated;) therefore [b] all the ſides are =, and by conſequence [c] the Angles. *Q. E. D.*

[z] *Ax.* 4.
[a] *Sup.*
[b] *Ax.* 1.
[c] *XVI.*

Theorem XXII.

Triangles [a b c, α β γ] *that have two Angles,* [α a, *and* β b,] =, *are equiangular; that is, have all their Angles* =.

WE are to prove that the Angle C = γ. For the three Angles A B C together are = [d] (2 ∟ = [d]) α β γ together. Therefore if from theſe two equal ſumms, you take away A = [e] α and B = [e] β, there will remain [f] C = γ. *Q. E. D.*

[d] *X.*
[e] *Sup.*
[f] *Ax.* 8.

Theorem XXIII.

In two Triangles, where one side, [a b, $\alpha\beta$] *and two Angles* [a, α, *and* c, δ] (*though not adjoyning to this side*) *are* =; *all are equal.*

THree Angles of S = [g] (2 ∟ = [g]) three Angles of O together. If now from these equal summs you take away equals, *viz.* A = [h] α and C = [h] δ, there will [i] remain B = β; therefore [k] (the side A B being = [h] $\alpha\beta$) all are =. *Q. E. D.*

g *X.*
h *Sup.*
i *Ax.* 8.
k *XXI.*

Theorem XXIV.

In two Triangles [a b c, d e f] *where two sides are* =, *and one Angle* [a = d] (*though not between those sides* (*v. XVII,*) *all are* =; *provided that the other Angles be of the same kind*; viz. ∟, *acute, or obtuse Angle.*

BEcause the Angle D = A, therefore being laid upon it, they will meet,[l] and the point E will fall on B (because the line D E = [m] A B,) also the point F, on C: if not, F must fall either below or above, in G or γ. First, not above, for let the Angle F be ∟ or obtuse, then A C B must [n] be the same, which is = [n] B γ C; (for B γ *i.e.* E F,

l *Ax.* 4.
m *Sup.*
n *XIII.*

E F, =[m] B C:) therefore A C B and B γ C will be both ⌊ or obtuſe, which is contrary to Th. X. 2*dly*, Nor can F fall below, for let F, that is G, be ⌊ or obtuſe, it will be =[n] to B C G (becauſe B G (*i.e.* E F) =[m] B C) that is, 2 ⌊ or obtuſe Angles in one Triangle B C G, contrary to X. *Laſtly*, Let the the Angle F be acute, A C B muſt[m] be the ſame, therefore B C G is obtuſe =[n] B G C, contr. X. Or (above) let A γ B, *i.e.* F, be acute; therefore B γ C is obtuſe =[n] B C γ, contr. X: therefore E F will fall on B C. And the three ſides being =, [o] all will be =. *Q. E. D.* [o] *XVI.*

Theorem XXV.

Of ſeveral lines that can be drawn from a point given [c], *to a Line*, [a b] *the ſhorteſt is a perp.* [c d] *And of the reſt, the nearer to this the ſhorter.*

BEcauſe in the Triangle C D E the Angle D is ⌊ [p] (and ⊏ [q] then either of the others) therefore the ſubtendent C E is ⊏ C D. Further, becauſe C E D [q] is acute, therefore [r] C E F is obtuſe, and thereupon [ſ] C F ⊏ C E. *Q. E. D.*

[p] *Sup.* [q] *X.* [r] *I.* [ſ] *XV.*

Theorem XXVI.

Triangles [c a e, c b e] *upon the ſame baſe* [c e] *and of the ſame height* (*def* 47) (*or, which is the ſame, between the ſame Parallels* [a b, c d],) *are* =.

DRaw E F and B D Parallel to A C, becauſe the Angle { C E A = [t] E A F / C A E = [t] F E A } and the ſide A E is common, therefore [u] the Triangle C A E = E A F. In like manner the Triangle E F B, = E D B and C D B = C A B. Therefore A C E + E D B (½ of the whole, A C D B) = E D B + C B E; and taking away E D B which is common to both, there remains [w] A C E = C B E. *Q. E. D.*

[t] *IV.*
[u] *XXI.*
[w] *Ax.* 8.

Theorem XXVII.

Lines [a b, c d] *are Parallels, if equal Triangles* [c a d, c b d] *upon the ſame baſe* [c d] *can ſtand between them.*

IF C D be not parallel to A B, then the Parallel muſt fall either above or below A B, as in F. Draw out C B to F, and joyn F D. Therefore the triangle C F D = [x] (C A D = [y]) C B D. *Q. E. A.*

[x] *Præ.*
[y] *Sup.*

Theorem

Theorem XXVIII.

Triangles [S X] *upon* = *baſes* [e f. e d], *between the ſame Parallels* [a b. c d], *are* =.

LEt the triangles S and X be ſo plac'd, that A B may be = C F or E D, then joyn A E and B F. Now becauſe C F = A B, and A F is common, and the Angle A F C = [z] F A B, therefore [a] the Triangle S = A F B; in like manner [x] = A E B, and A E B = [b] A F B; ſo then S = (A F B = A E B =) X. *Q. E. D.*

[z] *IV.*
[a] *XVII.*
[b] *XXVI.*

Theorem XXIX.

Thoſe Lines [a b, c d] *are Parallel, which have between them equal Triangles* [c a e, e b d] *ſtanding on equal baſes.* [c e, e d]

FOr if A E be not Parallel to E D, the Parallel will fall either above or below; which cannot be, as in XXVII; therefore &c. *Q. E. D.*

Theorem XXX.

A Parallelogram [abcd] *is divided in the middle by the Diameter* [cb] *and the oppoſite ſides are equal.*

[c] *IV.* [d] *XXI.*

BEcauſe the Angle $\left\{\begin{matrix} CBA = {}^{c} BCD \\ BCA = {}^{c} CBD \end{matrix}\right\}$ and the line CB is common, therefore [d] the Triangle CAB = CBD, and by conſequence the Pgr. is divided into two = parts. And becauſe of the = Triangles, the ſide AB = CD and AC = BD. *Q. E. D.*

Note, The oppoſite Angles [*a d*, and *c b*] of a Pgr. are =.

Since the two Triangles ABC and CBD are =, therefore [e] the reſpective Angles are =, *viz.* A = D, and the 2 at C = 2 at B, *Q. E. D.*

[e] *XVI.*

Theorem XXXI.

Two Diagonals [ad, cb] (*or Diameters*) *in a Pgr. cut themſelves in the middle* [e].

[f] *IV.* [g] *XXX.* [h] *XXI.*

BEcauſe in the Triangles AEC and BED the Angle EBD = [f] ECA, and EDB = [f] EAC, and the ſide A = [g] BD, therefore [h] the Triangles are =, and by conſequence the ſide AE = ED, and BE = EC. *Q. E. D.*

Theorem

Theorem XXXII.

A Line [a b] *passing by the middle* [e] *of the Diameter* [d c] *cuts the Pgr. into two equal parts.*

BEcause O + Z = [i] X + S, and Z = [k] X, (for the Angle E C A = [l] E D B, and E A C = [l] E D B, and the side E C = [m] E D) therefore [n] taking away the equals Z and X, there remains O = S, which equals being added, it will be O X = S Z. *Q. E D.*

[i] *XXX.* [k] *XXIII.* [l] *IV.* [m] *Sup.* [n] *Ax.* 8.

Theorem XXXIII.

In a Pgr. the Complements (*so S and Z are call'd*) *are* =.

FOr O Z A = [o] X S V, all together. Also, O = [o] X and A = [o] V, therefore [p] there remains Z = S. *Q E. D.*

[o] *XXX.* [p] *Ax.* 8.

Theorem XXXIV.

Pgrs. [a d, c e, f g] *between the same Parallels* [a f, c h] *upon the same* [c d], *or equal* [c d, g h] *bases, are equal.*

WE are to prove that A D = C E = F G, thus, The Triangle C B D ([½] [q] A D and

[q] *XXX.*

of

q XXVIII. of CE) = q EGH (½ of FG.) Therefore
t Ax. 7. r the wholes, AD, CE and FG are, also equal.
Q. E. D.

CHAP. II.

Of a Circle. PART. II.

Theorem XXXV.

A Line [b d] *passing the Center, is perpendicular to a Chord* [a c] *which it divides in the middle* [d].

s *Radius's Def.* 21. f *Sup.* t *XVI,* u *I.*

FOr the side EA = s EC } and DE is com-
DA = f DC }
mon, therefore t the Angles at D are =, and by consequence u ⌊. *Q. E. D. (Def.* 7.)

Theorem XXXVI.

And if it [b d] *be perpendicular, it divides the Chord* [a c] *in the middle.*

w *Radius's.* x *Sup.* y *XXIV.*

FOr the side EA = w EC, ED is common, and the Angles at D are = x, therefore AD = y DC. *Q. E. D.*

Theorem

Theorem XXXVII.

f it [b d] *divides a Chord* [a c] *in the middle, and be alſo Perpendicular to it, it paſſes through the Center.*

F not, let E D paſs thongh the Center, therefore the Angle E D C [z] is ∟ = [a] B D C that , a part = to the whole. *Q. E. A.*

[z] *XXXV.*
[a] *Sup.*

Theorem XXXVIII.

That point [a] *is the Center of a Circle, from whence more than two equal* [a b, a d, a c] *right Lines can be drawn to the Circumference.*

Draw the Lines B D, D C divided in the middle by A E, A F, becauſe the ſides B E = [b] E D, A B = [c] A D, and A E is common, therefore he Angles at E are =, and by conſequence [d] ∟; herefore [e] E A paſſes the Center. As in like anner, by the ſame reaſon does F A. Since herefore both theſe paſs the Center, It muſt be where they meet; *viz.* in A. *Q. E. D.*

[b] *Conſtruct.*
[c] *Sup.*
[d] *I.*
[e] *Pre.*

Theorem

Theorem XXXIX.

Crossing Chords [a c, b d] (*not passing* [illegible] *Center*) *do not cut each other in* [illegible] *middle.*

FOr if E were the middle of both, then [illegible] (passing the Center) would make F E C
[e] *XXXV.* [f] to A C, and F E D ⌊ [f] to B D; that is F E
[g] *Ax.* 2. = [g] F E D a part = to the whole. *Q. E. A.*

Theorem XL.

Of several Lines [a c, a d, a e] *dra* [illegible] *from one point* [a] (*in a Circle*) *to* [illegible] *circumference, the greatest* [a c] *pa* [illegible] *the Center; the rest are* ⊏ *as nearer* [illegible] *this.*

[h] *Radius's.* 1. B C = [h] B D, therefore A B C (= [i] A B
[i] *Ax.* 8. ⊏ [k] A D. *Q. E. D.*
[k] *XI.* 2. B F D ⊏ [k] B D (= [h] B F E, and omitti
the common B F) F D ⊏ [i] F E; and (adding t
common A F) A D ⊏ [i] (A F E [k] ⊏) A
Q. E. D.

Theore

Theorem XLI.

Several Lines [a b, c d, a e] *drawn from one one point* [a] (*without a Circle*) *to the inner circumference* [b d e] *the greateſt* [a b] *paſſes the Center; the reſt are greater as nearer.*

A C + (C D = [l]) C B ⊏ [m] A D. *Q. E. D.*

2. C F D ⊏ [m] (C D = [l]) C F E; therefore (omitting the common C F) F D ⊏ [n] F E, and (adding the common F A) D F A ⊏ [n] (E F A [m]) E A. *Q. E. D.*

l *Radius's.*
m *XI.*
n *Ax.* 8.

Theorem XLII.

Several Lines [a b, a d, a e] *drawn from one point* [a] (*without the Circle*) *to the outward circumference,* [b d e] *the leaſt* [a b] *is that which being continued would paſs the Center, the reſt are leſs as nearer to this.*

C D A ⊏ [o] C B A, therefore (omitting the equals [p] C D, C B) D A ⊏ [q] B A. *E. D.*

o *XI.*
p *Radius's.*
q *Ax.* 8.

2. Draw

o *XI.* 2. Draw out C D to F, C E F ⊏ o C D
p *Radius's.* and (omitting the equals p C E, C D) E F ⊏
D F, to which adding the common F A, E F
q *Ax.* 8. ⊏ q (D F A ⊏ o D A. *Q. E. D.*

Theorem XLIII.

If one Circle touches another, a li
[b a] drawn through both their Cent
[b f] will fall upon the point of tou
ing [a].

r *Radius.* IF not, let the Center of the leſſer O be C, a
s *Ax.* 8. the line that paſſes through both the Cent
t *XI.* B, C, fall on D. Since then C is the center of
u *Radius's of the* ⊏ O, C A = r C E, to which if you add
O. the common B C, B C E (= s B C A) ⊏ t
A = u) B D. *Q. E. A.*

Theorem XLIV.

If Circles touch without, a line [b c] t
joins their centers will paſs through
point of touching [o.]

IF O be not the point of touching, let it be
then B A, C A will be Radius's of their
ſpective Circles; and by conſequence = to B
w *XI.* O C ⊐ w B A, A C, which is a contradicti

Theo

Theorem XLV.

Circles whether within or without, touch but in one point [a].

IF it be poſſible, let them touch alſo in another point, as B, draw the Line E A through the Centers E F; therefore F A = [x] F B, and (adding the common E F) E F A (= [y] E F B) ⊏ [z] E B, againſt *def.* 21.

Alſo if they touch without (as in *fig.* XLIV) in O and A, then B A, A C = B O, O C, contrary to *XI*; wherefore they touch but in one point. *Q. E. D.*

x *Radius's.*
y *Ax.* 8.
z *XI.*

Theorem XLVI.

One Circle cuts another in two points only.

IF it be poſſible let them cut in four, A B C D, therefore four Lines drawn from theſe points to the Center of the O *α*, will be equal [a]. Whereas by *XLII*, thoſe will be ⊏ which are neareſt the Center of the O *β*. If the Centers fall within both Circles, it will be in like manner contrary to *XLI*.

a *Radius.*

Theorem

Theorem XLVII.

A perpendicular [b c] *to the end of the Diameter* [b,] *falls all of it without the Circle.*

FRom the Center D, draw the Secant D C, the Angle D B C is ∟ [b]. and thereupon ⊏ [c] D C B; therefore the ſubtendent D C ⊏ [d] D B, a Radius, and by conſequence the point C is without the Circle. The ſame reaſon will hold for all points between C and B, therefore *&c. Q. E. D.*

[b] *Sup.*
[c] *X.*
[d] *XV.*

Theorem XLVIII.

The Angle [d a b] *of the Radius* [d a] *and Circumference is* ⊑ *than any acute Angle* [d a e.]

DRaw D E perpendicular to A B; the Angle D E A is ∟ [e], and ⊏ [f] D A E. Therefore the ſubtend. D A ⊏ [g] D E, ſo that D E muſt fall within the O, and by conſequence the Angle D A E is but a part of D A B, and ſo ⊐ than it. *Q. E. D.*

[e] *Conſtr.*
[f] *X.*
[g] *XV.*

Theorem

Theorem XLIX.

The Tangent [b a] *of a Circle (def.* 27.) *makes a* ∟ *with the Radius* [c a].

FOr if it made an acute, it would fall within the Circle ([h] since the Angle of the Circumference and Radius is ⊏ than any acute,) (*against* def. 27.) if an obtuse, then the Angle on the other side C A D, would be acute; and by [h] consequence wou'd fall within the Circle, against def. 27.

[h] *Pre.*

Theorem L.

Two Tangents [a b, a c,] *drawn from the same Point* [a], *are equal.*

FOr the Angles, D C A and D B A are ∟ [i], and thereupon [k] =. Also D C B = [l] D B C, (for D B [m] = D C,) which being taken from the two foresaid ∟ Angles; there [n] remains A B C = A C B, and by [o] consequence the subtendent A B = A C. *Q. E. D.*

[i] *Pre.*
[k] *Ax.* 2.
[l] *XIII.*
[m] *Radius.*
[n] *Ax.* 8.
[o] *XII.*

Theorem LI.

Lines, [a d, b c] [p] *equidistant from the Center* [e] *are equal.*

[p] *Def.* 33.

LEt fall the Perpendiculars E F, E G, from the Center E.

The side $\left\{\begin{matrix} EA \overset{q}{=} EB \\ EG \overset{p}{=} EF \end{matrix}\right\}$ and the Angle E G A, ($\sqsubset$ [r]) $=$ [s] E F B. [t] Therefore the Triangle A E G $=$ B E F, and the side A G $=$ B F. In like manner the side G D $=$ F C: therefore the whole A D [u] $=$ F C.

[q] *Radius's.* [r] *Constr.* [s] *Ax.* 2. [t] *XXIV.* [u] *Ax.* 7.

Theorem LII.

Of Lines [a b, a c, a d,] *in a Circle, the greatest is the Diameter* [a d,] *the rest are greater as nearer to this.*

1. A E D ([w] $=$ A E C) $\sqsubset$ A C. *Q. E. D.*

2. The Angle A E C $\sqsubset$ [x] A E B, therefore the subtend A C $\sqsubset$ [y] A B, (for the two sides A E C $=$ A E B, all Radius's) *Q. E. D.*

If it be said that F G, is further from the Center than A C, and yet not $\urcorner$ A C, from the point A draw a Line (as A B) equidistant from the Center with F G, which shall be $=$ [z] to F G but $\urcorner$ than A C, as before.

[w] *AD, AC, being Rad. and A E, common.* [x] *Ax.* 5. [y] *XX.* [z] *Pre.*

Theore

Theorem LIII.

The opposite Angles [b d, a c,], *of a four sided Figure* [a b d c] *inscrib'd in a Circle, are equal to* 2 ∟.

DRaw the Radius's, LA, LB, LC, LD; the several Angles subtended by these Radius's will be [a] =, *viz.* [a] *XIII.*

$$\left.\begin{matrix}E\\F\\I\\K\end{matrix}\right\} \overset{a}{=} \left\{\begin{matrix}G\\H\\M\\N\end{matrix}\right\}$$

Now all these Eight together, are = 4 ∟ ([b] because the Figure ABCD, may be divided into two Triangles) therefore either half of them, EFIK, that is BD; or GHMN, that is, AC, are = 2 ∟. *Q. E. D.* [b] *X.*

Part II.

In case the four sided Figure inscribed, (def. 90.) falls without the Center [i].

DRaw the Radius, IA, IC, IF, IH, to the several Angles. Now in the Triangle AIH, the Angle A[c] = H; and these Angles also are =, *viz.* [c] *XIII.*

[c] *XIII.* [d] *X.*

$$\left.\begin{matrix}\left\{\begin{matrix}A\\+\\B\end{matrix}\right.\\E\\F\end{matrix}\right\}^{c}=\left\{\begin{matrix}C\\D\\G\\+\\H\end{matrix}\right.$$

Omitting therefore the equals A and H, there remain equal halvs, BEF, and DGH, which altogether, are $= 4\llcorner$ ([d] because the whole Figure BCFG, may be divided into 2 Triangles,) and therefore either half, (being the opposite Angles,) are $=$ to $2\llcorner$. *Q. E. D.*

Theorem LIV.

All Angles [a, e] *in the same Segment,* [b f d] (*def.* 30.) *are equal*

[e] *Pre.* [f] *Ax.* 8.

$$\left.\begin{matrix}C+A=\\C+E^{e}=\end{matrix}\right\}2\llcorner.$$

Therefore (omitting the common C) $A^{f}=E$. *Q. E. D.*

Theorem LV.

An Angle [c f d] *in a Segment* [c e d] *at the Center* [f], *is double to that* [c b d] *which is at the Circumference.*

[g] *IX.* [h] *XIII.* [i] *Radius's.*

FOr the Angle $CFD^{g}=FDB+B$; which $FDB^{h}=B$, (the subtend, FD being $^{i}=FB$) and $B=A$: Therefore CFD is double to B or A. *Q. E. D.*

Theorem

Theorem LVI.

The Angle [a b c] *in the Semicircle is* ∟.

THe Angles A, B, F + E, D, C [k] = 4 ∟. But F + D [l] = 2 ∟: Therefore A, B, E, C [m] = ∟. But B, E, [n] = A, C, that is, either of these pair are = ∟, and by consequence B + E, the Angle at the Semicircle is ∟. *Q. E. D.*

[k] *X.*
[l] *I.*
[m] *Ax. 8.*
[n] *XIII.*

Theorem LVII.

An Angle in a Segment { ⊏ ⊐ } *the Semicircle is* { *Acute*, *Obtuse*.

BEcause CBD is [o] ∟, therefore CBA is [p] obtuse; and CBE, [p] acute. *Q. E. D.*

[o] *Pre.*
[p] *Def. 8, 9.*

Theorem LVIII.

Equal Angles, [b, e; k, l;] *whether at the Center or Circumference, are subtended by equal Arches,* [a c, d f.] *of* = *Circles.*

BEcause the Angle E [q] = B, and the sides DE F [r] = ABC, therefore being laid upon

[q] *Sup.*
[r] *Radius's of* = ○s.

one another, they ſhall [f] meet, and by conſequence the points D, F with the points A, C; likewiſe the ◯s being [q] =, (if the Center E be laid upon the Center B,) they ſhall meet, and by conſequence the Arch D F with A C; wherefore they are =. *Q. E. D.*

Now K, L, are ſubtended by the ſame Arches which are already prov'd =.

[f] *Ax.* 4.
[q] *Sup.*

Theorem LIX.

The Angle [a e f] *at the Center, ſtanding upon half the Arch* [a f], *is* = *to the Angle at the Circumference, ſtanding upon the whole Arch* [a f c].

FOr the Angle A E F [t] = C E F, (the Arch A F being [u] = F C) but A E F is [w] double to A D F, and C E F is [w] double to C D F: Therefore A D F [x] = C D F, and both together (*viz.* A D C) are = A E F; but A D C [y] = A B C, therefore [z] A B C = A E F. *Q. E. D.*

[t] *Pre.*
[u] *Sup.*
[w] *LV.*
[x] *Ax.* 7.
[y] *LIV.*
[z] *Ax.* 3.

Theorem

Theorem LX.

A Line [d c] *passing through a Circle at the Point* [c] *of touching, makes an Angle to the Tangent* [a b] = *to an Angle in the opposite Segment, viz.* DCA = E, *and* DCB = M.

1

2

DRaw the Diameter F C, and joyn F D.

1. Because the Angles [a] FDC, and [b] FCA, are ∟. Therefore K [c] + H, or DCA + H [d] = ∟, and (omitting the common H) there remains DCA [e] = (K [f] =) E. *Q. E. D.*

2. The Angle E + M [g] = (2 ∟) [h] = DCA + DCB: Therefore (taking away equals, DCA [i] and E,) there remains DCB = M. *Q. E. D.*

[a] *LVI.*
[b] *XLIX.*
[c] *X.* [d] *Ax.* 5.
[e] *Ax.* 8.
[f] *LIV.*
[g] *LIII.*
[h] *L.* [i] *I.*

CHAP. III. PART I.

Of Proportion in General.

Theorem LXI.

If four Terms (A . B :: C . D) *be Proportional, all the following* Changes *of the ſaid terms ſhall be likewiſe Proportional.*

	A . B :: C . D	Let { A . B :: C . D / 12 . 9 :: 8 . 6
1. *Inverted.*	B . A :: D . C	9 . 12 :: 6 . 8
2. *Alterned.*	A . C :: B . D	12 . 8 :: 9 . 6
3. *Compounded.*	A+B . B :: C+D . D	21 . 9 :: 14 . 6
Or.	A+C . C :: B+D . D	20 . 8 :: 15 . 6
4. *Divided.*	A−B . B :: C−D . D	3 . 9 :: 2 . 6
Or.	A−C . C :: B−D . D	4 . 8 :: 3 . 6
5. *Converted.*	A . A ± B :: C . C ± D	12 . 21 :: 8 . 14 12 . 3 :: 8 . 2
Or.	A . A ± C :: B . B ± D	12 . 20 :: 9 . 15 12 . 4 :: 9 . 3
6. *Mixed.*	A+B . A−B :: C+D . C−D	21 . 3 :: 14 . 2
Or.	A+C . A−C :: B+D . B−D	20 . 4 :: 15 . 3

That

That all these Changes are Proportional, may appear sufficiently plain from the numbers annext according to the following instances, in Theor. LXII. only Note by the way, that all these Changes of the Letters, agree also to the sides of the Triangle, (Fig 61.) which it would not be amiss to look upon all the way, as you read these over. *Note* 1.

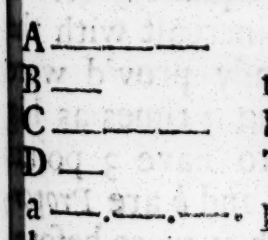
2. The first Change, *Inverted*, affirms no more than this, that if A be as big in respect of B, as C is in respect of D, then B is as little in respect of A, as D is in respect of C; which may pass for an Axiom. 2.

Theorem LXII.

The second Change, Alterned $\left\{ \begin{array}{l} A.C :: B.D \\ 12.8 :: 9.6. \end{array} \right.$

SUppose A, B, C, D; to be Lines divided into so many parts, *viz.* 12, 9, 8, 6; then 12 . 8 :: 9 . 6, for 8 is contained once and a $\frac{1}{2}$ (*viz.* 4.) in 12, and 6 once and a $\frac{1}{2}$ (*viz.* 3.) in 9. So that B and D are like parts of A and C, and by consequence A . C :: B . D. *Q. E. D.*

The same method will make all the rest of the Changes appear proportional.



But in order to a further Demonstration of this thing, (if it be required) let us consider, That the first Axiom about Proportion, which is naturally evident, and on which the whole Doctrine

[k] *Ax.* 10. *Fig. LXII.* Doctrine of it is founded, is this, [k] That equal things (A, C,) have the ſame bigneſs in reſpect of a third; (or of equals (B, D). Since in this conſideration equals are but the *ſame*, (for 20 Yards in a bundle, make but one Yard meaſure.) In this caſe, (A . B :: C . D) Proportion and Equality is coincident; the Terms being both Equal and Proportional.

Let us now increaſe the two firſt of theſe equal terms, (A and C). And to do it by degrees, (that the way may be clear,) let us firſt add one point to the meaſurer B, (as in *b*) (the effect of which is this; that A is not now ſo big in re-

[l] *Def.* 39. ſpect of *b*; as C is, in reſpect of D. $\left(\frac{A}{b} \sqsupset \frac{C}{D}\right)$ and thus the Proportion is deſtroy'd: to reſtore which) by conſequence we muſt in *Proportion*, add 3 points to the meaſured A; (as in *a*)

[m] *Sup.* [m] becauſe A is 3 times as big as B. Thus the Proportion is reſtored, (and the equality deſtroyed) *a* being now three times as big as *b*; juſt as C is to D. (*a* . *b* :: C . D). *Q. E. D.*

To Proceed.

It is naturally evident, in the firſt 4 terms, that A is as big in reſpect of C; as B is, in reſpect of D, (becauſe of their equality) (A . C :: B . D) conſider them now increaſt with the points: which we have already prov'd were added *Proportionably*; (A being 3 times as big as B; and therefore ought to have 3 points for the others one,) ſo that *a* and *b* are *Proportionably* increaſt above what they were before

tha

that is, *a* is as much bigger than it was (A); as *b* is than it was (B). Or (ſince A, C and B, D are equal) *a* is much bigger than C, as *b* is than D; *a*. C :: *b*. D, which is *Alterned*. *Q. E. D.*

And thus how many points ſoever (or Lines) you add to B, if you add of the ſame to A, in *Proportion* to its bigneſs above B, there will ſtill be the ſame reaſon why *a* ſhould be to C, as *b* is to D. For ſince A and B were [n] Proportional to C and D; and are now Proportionally increas'd (in *a* and *b*) above what they were before, that is above C, D, it follows that they muſt ſtill be Proportional to C, D; that is, *a*. C :: *b*. D. *Q. E. D.*

[n] *being equ.*

Theorem LXIII.

Like Parts are to one another, as their Wholes are.

B and D are like parts of the whole ſides of the Triangle, it being prov'd in LXI. 3. that A+B . B :: C+D . D, 21 . 9 :: 14 . 6, therefore *Alternedly*, (LXI. 2.) it ſhall be, A+B . C+D :: B . C. 21 . 14 :: 9 . 6. *Fig. LXI.*

Theorem

Theorem LXIV.

Reasonable Proportion. 1. *In Order.*

If in continued Proportion there be several terms in two rows, A,B,C, ∴ D,E,F, &c. ∴ *Or if several terms be carried on proportionaly, two in one row to two in another*, A . B :: D . E, and B . C :: E . F, &c. *then the extreams* (A, C) *in one row, shall be proportional to the extreams in the other.*

If,

A, B, C, &c. ∴ D, E, F, &c. ∴

Or if,

A	.	B	::	D	. E
12	.	6	::	18	. 9
B	.	C	::	E	. F
6	.	4	::	9	. 6
&c.				&c.	

Then,

A	.	C	::	D	. F
12	.	4	::	18	. 6

AS may appear from the numbers, becauſe $\frac{A}{12}$ is contain'd in $\frac{C}{4}$ three times, juſt as $\frac{F}{6}$, is in $\frac{D}{18}$. But for further proof, $\frac{A}{C}$ may be call'd

a

a whole; as being compoſed of $\frac{A}{B}$ and $\frac{B}{C}$. The like of $\frac{D}{F}$ as compos'd of $\frac{D}{E}$ and $\frac{E}{F}$. But the parts of both theſe wholes are $=$ (*viz.* $\frac{A}{B}$ ° $=$ $\frac{D}{E}$ ° *Sup.* and $\frac{B}{C}$ ° $=$ $\frac{E}{F}$) and therfore the wholes ($\frac{A}{C}$ & $\frac{D}{F}$) muſt be $=$ too, (*Ax.*5.) *Q. E. D.*

Theorem LXV.

If the former terms remaining, you prefix S *to the ſecond row, ſo that the two laſt* (B, C) *of the firſt row, may be proportional to the two firſt* (S,D) *of the ſecond row, it ſhall be as* A . C :: S . E. 2. *Diſturb'd*

S.

A, B, C, ∴ D, E, F, ∴

Or if,

S.

27

A . B :: D . E

12 . 6 :: 18 . 9

B . C { :: E . F

6 . 4 { :: S . D 9 . 6

27 . 18

Then,

A . C :: S . E

12 . 4 :: 27 . 9

For

P *Sup.* For, $\frac{E}{F}{}^{P}=\left(\frac{B}{C}{}^{P}=\right)\frac{S}{D}$. Therefore by Alter-

q *Pre.* ning and Inverting $\frac{S}{E}=\left(\frac{D}{F}{}^{q}=\right)\frac{A}{C}$. *Q. E. D.*

It may appear also from the numbers.

Theorem LXVI.

Unproportional terms, are unproportional in all the foregoing Changes.

Fig. LXII. If $\frac{A}{B}\sqsubset\sqsupset\frac{C}{D}$: Then also Alterned, &c. it shall be $\frac{A}{C}\sqsubset\sqsupset\frac{B}{D}$. For suppose the one point only added to B (as in *b*,) A is now not so big in respect of *b* as C is in respect of D (as we observed) $\left(\frac{A}{b}\sqsupset\frac{C}{D}\right)$ and by consequence *Alternedly* $\frac{A}{C}\sqsupset\frac{b}{D}$; for A is but = C, but *b* is ⊏ D, by a point. All which is naturaly evident, by looking on the Lines, and may easily be applied to the other Changes. To give one instance in numbers, let it be $\frac{12.A}{9.B}\sqsupset\frac{C.8}{D.5}$ then Alterned, $\frac{12.A}{8.C}\sqsupset\frac{B.9}{D.5}$. For, 8 is contain'd but once and a ½ (*viz.* 4.) in 12, whereas 5 is contain'd almost twice in 9: So that 9 is bigger in respect of 5 than 12 is, in respect of 8, or, what is the same, 12 is ⊐ in respect of 8, &c.

CHAP

CHAP. III. PART II.

Of Proportion of Triangles, *&c.*

Theorem LXVII.

The ſides [$\alpha\beta, \alpha\gamma$] *of a Triangle are cut Proportionally by a Line* [$\eta\theta$] *Parallel to the baſe* [$\beta\gamma$], *viz.* A . B :: C . D.

THe Line $\delta\varepsilon$ being always mov'd Parallel towards $\beta\gamma$, will at laſt meet with it in all points at once (for if any part of $\delta\varepsilon$ ſhould touch before another, it would have been inclin'd in that part to $\beta\gamma$, and by conſequence would not be Parallel) and by conſequence in the points β and γ. Since then $\delta\varepsilon$ begins at once upon both the ſides of the Triangle $\alpha\beta$, $\alpha\gamma$; and at the ſame time comes to the end of both, β and γ, it follows, that when it is come to the middle of one, it is alſo at the middle of the other; when it has paſt over three parts of one, it has paſt the like of the other, *&c.* So that B, D, will be always *like* parts of the ſides A + B and C + D, and by conſequence A + B . B :: C + D . D, and by diſſolving the compoſition, A . B :: C . D. *Q. E. D.*

Theorem

Theorem LXVIII.

A Line [$\eta\theta$] *cutting the ſides of a Triangle Proportionaly, is Parallel to the baſe* [$\beta\gamma$].

FOr if any part of $\delta\varepsilon$ (that is $\eta\theta$) ſhould touch the baſe $\beta\gamma$ before another, it would be inclin'd to the baſe in that part, and by conſequence would not be Parallel. *Q. E. D.*

Theorem LXIX.

The baſe is to the Parallel Line, [$\beta\gamma$] *as the ſides are to their parts next the top* [α]: *That is,* E + F . G :: B + A . A :: D + C . C.

FOr $\beta\delta$ being drawn Parallel to C D, it will be (by the Prec. *inverted*) D . C :: B . A
:: F . (E^{r} =) G therefore *compounded* (ſuppoſing C D the baſe, and η, the top) D + C . C :: B + A . A :: F + G . G. *Q. E. D.*

r XXX.

Theorem

Theorem LXX.

A Parall. to the baſe, cuts off a part [X], *towards the top*, Like [::] *to the whole Triangle.* *Def.* 45.

FOr the Angles $\left\{\begin{matrix}\alpha\gamma\beta \\ \alpha\beta\gamma\end{matrix}\right\}^{r} = \left\{\begin{matrix}\alpha\theta n \\ \alpha n\theta\end{matrix}\right\}$ and α is common, therefore X is equiangled to the whole Triangle. [r] *IV.*

Further, the ſide [s] DC. C :: FE. G :: BA. A: Therefore *Alterned*, DC. FE :: C. G and FE. BA :: G. A: [s] *Pre.*

Laſtly becauſe B. A :: [u] D. C, therefore *Compounded*, B + A. A :: D + C. C, and *Alterned* B + A. D + C :: A. C: Thus all the ſides about the equal Angles are proportional; and by [w] conſequence X is :: to the whole Triangle. *Q. E. D.* [u] *LXVII. Inverted.* [w] *Def.* 45.

Theorem LXXI.

Equiangled Triangles [$\alpha n\theta$, S.] *are* ::

CUt off, by the line $\beta\gamma$; $\alpha\beta, \alpha\gamma = \delta\varepsilon, \delta\zeta$, becauſe the Angle α [x] $= \delta$, therefore X $=$ S, therefore the Angle γ [y] $= (\zeta$ [x] $=)\theta$, therefore $\beta\gamma$ [z] Parall. $n\theta$; and by conſequence the Triangle $\alpha n\theta$, is [a] :: (X [b] ::) S. *Q. E. D.* [x] *Sup.* [y] *XVII.* [z] *VI.* [a] *Pre.* [b] *Ax.* 10.

Theorem LXXII.

Triangles [$\alpha n \theta$, S.] *are like, whose ſides are Proportional.*

TAke $\alpha\gamma = \delta\zeta$, and draw $\gamma\beta$ Parall. θn: Therefore $\frac{B+A}{A}$ [c] $= \left(\frac{D+C}{C \text{ [d] or } \delta\zeta}\right)$ [e] $= \frac{B+A}{\delta\varepsilon}$: therefore, A [f] $= \delta\varepsilon$: Further, $\frac{E}{F}$ [g] $= \left(\frac{D+C}{C \text{ [h] or } \delta\zeta}\right)$ [e] $= \frac{E}{\varepsilon\zeta}$: Therefore F [f] $= \varepsilon\zeta$; ſo then S has equal ſides to X, and by [h] conſequence = Angs. therefore is [i] :: (X [k] ::) $\alpha n \theta$. *Q. E. D.*

[c] *LXVII.*
[d] *Conſtr.*
[e] *Hyp. Alterned.*
[f] *Ax.* 10.
[g] *LXIX.*
[h] *XVI.*
[i] *Pre.*
[k] *LXX.*

Theorem LXXIII.

Triangles [$\alpha n \theta$, S.] *whoſe ſides are Proportional, are equiangled.*

THis is Demonſtrated in the operation of the Precedent; for the Suppoſition being the ſame in both, it is there prov'd, that S has = Angles to (X, which has [l] = Angles to) $\alpha n \theta$. *Q. E. D.*

[l] *LXX. and Def.* 45.

Theorem

Theorem LXXIV.

Triangles [$\alpha n \theta$, S.] *are* ::, *which have one Angle* =, [$\alpha = \delta$] *and the ſides about that Angle Proportional* B+A. C+D :: $\delta\varepsilon . \delta\zeta$.

CUt off, (by the Line $\beta\gamma$) $\alpha\beta, \alpha\gamma = \delta\varepsilon, \delta\zeta$. Becauſe the Angl $\alpha^{m} = \delta$, therefore $X^{n} = S$, and by [o] conſequence is ::, now becauſe A+B. ($\delta\varepsilon$ [p] or) A [m] :: C+D. ($\delta\zeta$ [p] or) C: Therefore $\beta\gamma$ Parallel $n\theta$, and by conſequence $\alpha n \theta$ is [r] :: (X ::) S. *Q. E. D.*

[m] *Sup.* [n] *XVII.* [o] *LXXI.* [q] *Conſt.* [q] *LXVIII.* [r] *LXVII.*

Theorem LXXV.

Triangles [$\alpha n \theta$, S.] *are* ::, *which have two ſides Proportional* [B+A. D+C :: $\delta\varepsilon . \delta\zeta$], *and the Angle oppoſite to theſe ſides,* = [$\theta = \zeta$] *and another of the ſame kind.*

CUt off (by the Line $\beta\gamma$) $\alpha\beta, \alpha\gamma = \delta\varepsilon, \delta\zeta$. Becauſe $\frac{B+A}{(\delta\zeta^{f}\text{or})A}$ [f] $= \frac{D+C}{(\delta\varepsilon^{t}\text{or})C}$: [u] Therefore $\beta\gamma$ Parallel $n\theta$. Now the Angle ζ [f] = (θ [w] =) γ: Therefore $X^{x} = S$, and by conſequence [y] ::, but $\alpha n \theta$ [z] :: (X ::) S. *Q. E. D.*

[f] *Sup. Alterned.* [t] *Conſtr.* [u] *LXVIII.* [w] *VI.* [x] *XXIV.* [y] *LXXI.* [z] *LXVII.*

Theorem LXXVI.

A Right-angled Triangle [a b c], *divided by a Perpendicular* [a d] *from the* ∟ *to the ſubtendent, is* :: *its parts* [a b d, a d c].

FOr each part has one Angle common to the whole, *viz.* B and C, and one Angle ∟ (*viz.* at D,) [a] = B A C: Therefore the whole is [b] equiangled to its parts, and by conſequence [c] is :: to them. *Q. E. D.*

[a] *Ax.* 2.
[b] *XXII.*
[c] *LXXI.*

Thorem LXXVII.

The ſides of a Triangle are Proportional to the parts of its baſe, divided by a Line that cuts the oppoſite Angle in the middle f g. A :: D. C.

DRaw out A, Infinitly, and G I Parallel I H, therefore the Angle F G I [d] = (H F G [e] = H F E [d] =) G I F, and therefore the ſubtendent F I [f] = F G. [g] But, (B that is) F G. A :: D. C. *Q. E. D.*

[d] *IV.*
[e] *Sup.*
[f] *XII.*
[g] *LXVII.*

Theorem

Theorem LXXVIII.

Triangles of the ſame height, (or between the ſame Parallels) are to one another as their baſes, that is, S . Z :: DC . CF.

TAke DB = CF, therefore X[h] = Z. Now if CA be mov'd, on the point A, as on a Center, towards AB, they will at laſt meet: and CA ſhall at the ſame time have paſt over the whole Triangle, (ABC,) and the baſe (BC;) and by conſequence when it has come to the middle of one, it will be at the middle of the other; and ſo, in Proportion, whatſoever part it has paſt over of the Triangle, it will have paſt over a like of the baſe; and (by conſeq.) will divide the Triangle and baſe Proportionally; that is, that S ſhall be to (X =) Z :: DC.(DB =)CF. *Q. E. D.*

[h] *XXVI.*

Theorem LXXIX.

Equal Triangles X, S, *having one* = *Ang.* [*at* C] *have their ſides about this Ang. reciprocally proportional,* AC . CE :: DC . CB.

LEt the = Angles ECD, BCA, be ſo joyned that EA, BD may be Right-lines, (which is

possible; as may easily appear from III. as the II. does from I. Then joyn BE: Now,

[i] *Pre.* [k] *For* X = S *Sup.*

$$\frac{AC}{CE}\ ^{i}=\left(\frac{X}{Z}\ ^{k}=\frac{S}{(Ax.10.)Z}\ ^{i}=\right)\frac{DC}{CB}.$$ *Q. E. D.*

Theorem LXXX.

Parallelograms [O, S,] *of the same height, are to one another, as their bases* [c d, d g].

[l] *XXX.* [m] *LXXVIII* [n] *Ax.* 7.

For the [l] Triangle CD, is $\frac{1}{2}$ O; also DFG, is $\frac{1}{2}$ S. But $\frac{CD}{\frac{1}{2}O}\ ^{m}=\frac{DG}{\frac{1}{2}S}$, and *Alterned*

$$\frac{CD}{DG}=\left(\frac{\frac{1}{2}O}{\frac{1}{2}S}\ ^{n}=\frac{\text{whole}}{\text{whole}}\right)\frac{O}{S}.$$ *Q. E. D.*

Theorem LXXXI.

Parallelograms [O, S,] *having one Angle* =, [*at*, c] *have their sides about this Angle reciprocally proportional*, EC . CA :: BC . CD.

[o] *Sup.* [p] *LXXII.* [q] *LXXIX.*

For since S[o] = O, therefore $\frac{1}{2}$ S (*viz.* the Triangle ECD) [p] = $\frac{1}{2}$ O, (*viz.* BCA:) and [q] therefore, EC . CA :: BC . CD. *Q. E. D.*

The orem

Theorem LXXXII.

Parallelograms [S, O,] *that have one Ang.* =, (*and by conſequence all Rectangles*) *are to one another in a reaſon* [r] *compounded of that of their ſides.* [r] *Def.* 43.

FOr ſince ſuch Figures have both Length and Breadth, to know the *reaſon* of one to the other, it is neceſſary to compare both their Length and Breadth, (that is, their ſides) and out of the *reaſon* of both theſe, is *compounded* the *reaſon* of the Figures. For inſtance, if S be twice as long, and thrice as broad as O, then S is two times thrice, (that is, 6 times) as big as O, (as may be ſeen by the prickt Lines, in S,) ſo that $\frac{S}{O} = \frac{DB}{BA} + \frac{EB}{BC}$. *Q. E. D.*

Theorem LXXXIII.

Like *Parallelograms* [S, O,] *are in a Duplicate* reaſon *to that of their Homologous ſides*, ÐB, BA, EB, BC.

THe reaſon of Pgrs. as we have ſaid, is (*compounded* or,) made up of the *reaſons* of their Length and Breadth. But now in *Like* Pgrs. the *reaſons* of their Length and Breadth,

are the ſame, (that is, S is juſt as much broader, as it is longer, than O:) So that, whereas in the former Propoſition, to find the reaſon of S to O, we were to joyn together the reaſons of their Length and Breadth; here it will be ſufficient to take the *reaſon* of either, twice; becauſe they are both the ſame: Thus, if S be twice as long, and (by conſequence) twice as broad as O, then it is two times twice (that is 4 times) as big as O, and this is call'd *Duplicate* reaſon. For Lines thus, $\frac{DB}{EB}$ [r] $= \frac{BA}{BC}$, and *Alterned*, $\frac{DB}{BA} = \frac{EB}{BC}$: But $\frac{S}{O}$ [t] $= \left(\frac{DB}{BA} + \frac{EB}{BC} =\right) 2\frac{DB}{BA}$ or, $2\frac{EB}{BC}$. *Q. E. D.* For further Illuſtration,

[r] *Sup.*

[t] *Pre.*

Fig. II. If the Length of S, be to the Length of O, as 4 to 3, by [u] conſequence the Breadth muſt be as 4 to 3 alſo. Then $\frac{S}{O}$ [w] $= \frac{4}{3} + \frac{4}{3}$, that is, $= 2\frac{4}{3}$: *viz.* $\frac{16}{9}$, as appears by the little Squares in S and O.

[u] *Def.* 42.

[w] *Pre.*

Theorem LXXXIV.

Parallelograms [S, O,] *are equal, which have one Angle* $\doteq$, [α, β,] *and the ſides about this, reciprocally proportional.*

Viz. If A . D :: C . B, then S = O. Now *Alterned*, A . C :: D . B; that is, S is as much

much longer than O; as O is broader than S, and [x] ſince the bigneſs of S to O, is made up of the bigneſs of the ſides, if S be 3 times longer than O; and O 3 times broader than S, theſe two reaſons in being *compounded* deſtroy one another, and S is left = O. *Q. E. D.*

[x] *LXXXII.*

Theorem LXXXV.

All Squares are :: (Figures, or) Pgrs.

FOr A[y] = B and C[y] = D, therefore, A . C :: [z] B . D, and *Alterned*; A . B :: C . D: Therefore S is [a] :: O. *Q. E. D.*

[y] *Sup.*
[z] *Ax.* 10.
[a] *Def.* 45.

Theorem LXXXVI.

Like Rect-angles [S, O,] *are to one another as the Squares of their Homologous ſides*, A . B.

FOr $\frac{S}{O}$[b] = (2 $\frac{A}{B}$[c] =) $\frac{Aq}{Bq}$. *Q. E. D.*

[b] *LXXXIII*
[c] *Pre. with LXXXIII.*

Theorem

Theorem LXXXVII.

F.LXXXII *Triangles that have one Angle* = , *are to one another in a reaſon compounded of that of their ſides.*

[d] *XXX.* FOr Triangles are [d] ½ Pgrs. (Diameters being drawn from E to D, and from A to C,) and likewiſe receive the ſame ſides with S and O.

[e] *LXIII.* [f] *LXXXII.* But $\frac{\frac{1}{2}S}{\frac{1}{2}O}$ [e] $= \left(\frac{S}{O}\right.$ [f] $= \left.\right) \frac{DB}{BA} + \frac{EB}{BC}$. *Q. E. D.*

Theorem LXXXVIII.

f.LXXXIII Like *Triangles, are in a Duplicate reaſon to that of their Homologous ſides.*

FOr theſe, again, are ½ the Pgrs. S and O

[g] *LXIII.* [h] *LXXXIII.* Therefore $\frac{\frac{1}{2}S}{\frac{1}{2}O}$ [g] $= \left(\frac{S}{O}\right.$ [h] $= \left.\right) 2\frac{DB}{BA}$ or $2\frac{EB}{BC}$

Q. E. D.

Theorem

Theorem LXXXIX.

Like *Pgrs.* [a d, e g] *plac'd* ::, *and having one common Angle* [c], *receive the ſame Diameter* [c b].

IF not, let the inner Pgr. be CFHG; and draw F G parallel D B: Therefore, the Angle C G F[i] = (D[k] =) C G H, (for C E H G is ſuppoſed :: to A D.) The like, if H fall on the other ſide the Diameter, therefore ſince H can fall on neither ſide the Diameter, it muſt fall upon it. *Q. E. D.*

[i] *IV.*
[k] *Def.*

Theorem XC.

Like *Figures* [a b c d e, f g h i k,] *of many ſides, may be divided into an equal number, of* :: *Triangles, by the Lines* [a c, a d, f h, f i].

THe firſt part is plain only by looking on the Figures; the ſecond, that the Triangles are ::, is prov'd thus: The Angle B = G; and the ſides, A B . B C :: [l] F G . G H: Therefore the Triangle A B C[m] like F G H; and by the ſame reaſon A E D like F K I. Laſtly C A D[n] like H F I; (for the whole Figures were [l] equiangled, and equal Angles have been taken away on [o] each ſide, therefore there remains = Angs. to C A D and H F I. *Q. E. D.* Theorem

[l] *Sup.*
[m] *LXXIV.*
[n] *LXXI.*
[o] *Being prov'd* ::

Theorem XCI.

Like *Figures* [a b c d e, f g h i k,] *are to one another, in a* Duplicate *reason (or as the Squares) of their Homologous ſides.*

For (every Triangle being [p] ½ a Pgr.) the [r] :: Triangles $\frac{ABC}{FGH}$ [q] $= 2\frac{AB}{FG}$ ([r] $= \frac{ABq}{FGq}$) (and $\frac{AED}{FKI} = 2\frac{AE}{FK}$ and $\frac{CAD}{HFI} = 2\frac{CD}{HI}$. But now, becauſe $\left.\begin{matrix} AB.BC \\ BC.CD \end{matrix}\right\} :: \left.\begin{matrix} FG.GH \\ GH.HI \end{matrix}\right\}$ therefore [t] AB . CD :: FG . HI; and by the ſame reaſon AE . CD :: FK . HI; and *Alterned* of both together $\frac{AB}{FG} = \frac{CD}{FI} = \frac{AE}{FK}$: Since then the reaſon of all theſe Homologous ſides, is equal, the Triangles $\frac{ABC}{FGH}$ or $\frac{AED}{FKI}$ or $\frac{CAD}{HFG}$ [u] $= 2\frac{AB}{FG}$) Therefore the wholes $\frac{ABCDE}{FGHIK}$ [w] $= 2\frac{AB}{FG} = \frac{ABq}{FGq}$. *Q. E. D.*

[p] *XXX.* [q] *LXXXIII.* [r] *LXXXV.* [s] *Pre.* [t] *LXIV.* [u] *Ax.* 10. [w] *LXIII.*

Theorem

Theorem XCII.

Circles are to one another in a Duplicate *reaſon of their Diameters, (or, as the Squares of Diameters* LXXXVI. *with* LXXXIII.)

THat is $\frac{\bigcirc,ADF}{\bigcirc,\alpha\delta\varphi} = 2\frac{AB}{\alpha\beta} = \frac{AB^q}{\alpha\beta q}$. If not, let $\frac{\bigcirc,ADF}{\bigcirc,\alpha\delta\varphi} \sqsubset 2\frac{AB}{\alpha\beta}$. And in the $\bigcirc$,ADF, let there be in inſcrib'd a Polygon (or many ſided Figure) Z in ſuch manner that $\frac{Z}{\bigcirc,\alpha\delta\varphi} \sqsubset 2\frac{AB}{\alpha\beta}$ (which appears to be poſſible, becauſe the ſides of the Polygon may be ſo multiplied till they come to be = Circumference of the $\bigcirc$ it ſelf, and by conſequence whatſoever the $\bigcirc$ exceeds, Z may likewiſe exceed.) Then in the $\bigcirc,\alpha\beta\varphi$, let the Polygon X be * inſcrib'd, like Z. Now the $\triangle$, AFB is [x] :: $\alpha\varphi\beta$; and by conſequence $\frac{AFB}{\alpha\varphi\beta} = [y]\ 2\frac{AB}{\alpha\beta}$ And. $\frac{Z}{X}$ [z] $= \left(\frac{AFB}{\alpha\varphi\beta}\right) 2\frac{AB}{\alpha\beta}$: But $\frac{Z}{\bigcirc,\alpha\delta\beta} \sqsubset$ [a] $2\frac{AB}{\alpha\beta}$. [b] Therefore X $\sqsubset \bigcirc,\alpha\delta\beta$. [c] *Q. E. A.* Therefore $\frac{\bigcirc,ADF}{\bigcirc,\alpha\delta\varphi} = 2\frac{AB}{\alpha\beta}$ or $\frac{AB^q}{\alpha\beta q}$. *Q. E. D.*

* *See Problem.*
x *XC.*
y *Pre.*
z *LXXIII.*
a *Conſtr.*
b *Ax.* 11.
c *Ax.* 5.

Theorem

Theorem XCIII.

All the external Angles of any Polygon are equal 2 ∟ *Angles.*

1. A Polygon may be divided into ſo many Triangles, as it has ſides, by Lines drawn from any point within, to the ſeveral Angles; this appears by looking on the Figure.

2. And by conſequence it contains twice as many [d] ∟ Angles, (as it has ſides;) except [e] 4 which are about the point in the middle, (for all the Angles about a point are = 4 ∟).

3. Then, drawing out the ſeveral ſides Each againſt the other makes 2 Angles [d] = 2 ∟; and by conſequence all the Angles together inward and outward, will be = to twice as many ∟, as there are ſides.

4. But the inward Angles are prov'd = to theſe, except 4, [f] and therefore the outward Angles remain = 4 ∟. *Q. E. D.*

[d] *X.*
[e] *I.*
[f] *XCIII*, 2.

Theorem XCIV.

Three only Regular *Figures (that is, whoſe ſides and Angles are =) can fill a ſpace* viz. *Triangles, as in* A, *Squares, as in* C, *and Hexagons, as in* B.

FOr ſince all the Angles about a point are [g] = 4 ∟. 1. Th

[g] *I.*

1. The Angle of a *regular* △ is [h] $= \frac{1}{3}$ 2∟ (becauſe all the 3 Angles are =) and by conſequence 6 of theſe joyn'd together, as in A, are = 4∟ Angles, for $\frac{6}{3}$ 2∟ = 4∟. [h] *X.*

2. The Angle of the □ is [i] ∟; and by conſequence the 4 at C are = 4∟. [i] *Def.*

3. The Angle of a Hexagon is [k] = 1 and $\frac{1}{3}$∟, and by conſequence the 3 joyn'd together in B are = 3 and $\frac{1}{3}$∟, that is, 4∟. [k] *Pre.* 2.

Now all Figures that have more ſides than this, have their Angles bigger than it, and by conſequence 3 of them (which is the [g] leaſt *(number)* that can make a plane Angle) will be ⊏4∟; In the Pentagon (of 5 ſides) 3 Angles will be ⊐, 4⊏ than [k] 4∟. *Q. E. D.*

Theorem XCV.

Like *Figures*, [S, O, Z, X,] *fram'd a like upon Proportional baſes*, [a. b :: c. d] *will be themſelves alſo Proportional*, viz. S. O :: Z. X.

FOr, $\frac{S}{O}$ [l] $= \left(\frac{A}{B} 2 \overset{m}{=} 2\frac{C}{D} =\right) \frac{Z}{X}$. *Q. E. D.*

[l] *XCI.*
[m] *LXIII, for A. B :: C. D, Sup.*

Theorem XCVI.

In a Circle, [b a c, b a d,] *the Angles are Proportional to the Arches* [b c, b d,] *that ſubtend them.*

FOr the Line A B, being mov'd towards C, at the ſame time will diſpatch both the Angle

A, and the Arch BC; and by consequence when it comes to the middle of one, it will be at the middle of the other, and so forward; so that in fine, whatsoever part BAD will be of BAC, BD will be the same part of BC: Therefore the Angle BAD, will be to BAC :: Arch B, to BC. *Q.E.D.*

Theorem XCVII.

A Polygon conscrib'd about a Circle is = to a Right Angled Triangle, whose base [ββ] *is the Compass of the Polygon, and its height* [αβ] *the Radius of the Circle.*

DRaw the Radius's AH, AI, &c. to the sides of the Polygon at the point of touching, then take βH = BH; and HC = HC; and so forward, 'till you come to NB; then draw αZ parallel ββ. Erect the Perpendiculars, HO, IP, KQ, &c. and joyn Oβ, OC; PC, PD, &c. Now, because the Angle OHB [n] = (∟ [o] =) αβH, therefore HO is [p] parallel to αβ; and by [q] consequence HO = (αβ [o] =) HA; therefore the △s, BAC and βOC have the same height; therefore BAC [r] = (βOC [f] =) βAC: In like manner CαD = (CPD =) CADᵢ &c. and, in fine, Gαβ = (GZB =) GAB. So then all the parts being equal, the whole △αββ [t] = to the Polygon. *Q.E.D.*

[n] *Constr.*
[o] *Sup.*
[p] *IV.*
[q] *XXX.*
[r] *XXVIII.*
[f] *XXVI.*
[t] *Ax.* 3.

Theorem

Theorem XCVIII.

Everry regular *Polygon is equal to a Right Angl'd* △, *whose base is the Compass of the Polygon, and its height, a Perpendicular* [a h] *to the side of the Polygon* [b c] *from the Center.*

FOr that Perpendicular *(a h)* will be the same with the Radius of the Circle that may be inscribed in such a Polygon, and than the rest follows from the Precedent.

Theorem XCIX.

A Circle is = *to a* ∟ *Angled* △, *whose base is the Circumference of the Circle, and its height, the Radius of it.*

1. EVery [u] Polygon conscrib'd is ⊏, and inscrib'd is ⊐, then the ◯. [u] *Ax.* 5.

2. The compass of a Polygon conscrib'd, is ⊏, and inscrib'd is ⊐, than the [u] circumference of the Circle.

3. This ∟ Angled △ will be [w] ⊐ than any Polygon conscrib'd, and ⊏ than any inscrib'd, (because the circumference of the ◯, which is the base of this Triangle is [x] ⊏ than the compass of any inscrib'd.) Therefore it will be = to the ◯. [w] *XCVII.* [x] *Ax.* 5.

For if this Triangle be ⊏ than any thing that is bigger than the ○, and ⊐ than any thing that is ⊐ than the ○, it follows that it must be equal to the ○. *Q. E. D.*

Note. And this is call'd the Squaring of a Circle, *viz.* to find a Right-lined figure equal to a Circle; upon this supposition, that the basis given is equal to the circumference of the Circle. But actually to find a Right-line = to the circumf. of a ○, is not yet discover'd Geometrically.

Theorem C.

An equal Legg'd Triangle [abc] *in a Segment, is* ⊏ *than* ½ *the Segment.*

DRaw the Radius FB Perpend. AC, and the Tangent DBE, which will be Parallel to AC, (becausе the Angle AGB[y] is ⌊ [z] = DBG) further draw AD and CE Parallel GB; DC will be a Pgr. and the Triangle ABC half of it; but the whole Pgr. is ⊏ than the Segment ABC; therefore ½ Pgr. *viz.* ABC, ⊏ [a] ½ the Segment. *Q. E. D.*

[y] *Const.*
[z] *XLIX.*
[a] *LXIII.*

Theorem

Theorem CI.

The Triangle DCB *is* ⊏ *than* ½ *the mixt Triangle* ACB.

LEt AB and DC be Tangents: Therefore DC = [b] DA; and Angle (DCF; [c] is ∟ [d] = DCB; therefore the ſubtend. DB [e] ⊏ (DC =) DA: Therefore the Triangle DCB [f] ⊏ (DCA [g] ⊏) ADC; the Triangle mixt, and by conſequence is more than ½ ABC the Triangle mixt. *Q. E. D.*

[b] *L.*
[c] *XLIX.*
[d] *I.*
[e] *XV.*
[f] *LXXVIII*
[g] *Ax.* 5.

From the two Preceding Propoſitions it appears, that figures inſcrib'd and conſcrib'd, if the number of their ſides be doubled, do approach the Circle by more than half the ſpace that was before left. *Note.*

Theorem CII.

A Circle contains more ſpace, than any other figure that is equal to a Circle.

FOr a ◯ is = [h] ∟ Angled △ whoſe baſe is the circumference of the ◯, and height, the Radius AB. But the Square CD (which we ſuppoſe = to the ◯) is [i] = to ∟ Angl'd △, whoſe baſe indeed, is the ſame with the former, (becauſe the compaſs of the Square is ſuppoſed = circumference of the ◯:) but the height is the Perpendicular AE. *Q. E. D.*

[h] *XCIX.*
[i] *XCVIII.*

Theorem XCIII.

All the external Angles of any Polygon are equal 2 ∟ *Angles.*

1. A Polygon may be divided into ſo many Triangles, as it has ſides, by Lines drawn from any point within, to the ſeveral Angles; this appears by looking on the Figure.

2. And by conſequence it contains twice as
[d] *X.* many [d] ∟ Angles, (as it has ſides;) except [e] 4
[e] *I.* which are about the point in the middle, (for all the Angles about a point are = 4 ∟).

3. Then, drawing out the ſeveral ſides
[f] *XCIII*, 2. Each againſt the other makes 2 Angles [d] = ∟; and by conſequence all the Angles together inward and outward, will be = to twice as many ∟, as there are ſides.

4. But the inward Angles are prov'd = to theſe, except 4, [f] and therefore the outward Angles remain = 4 ∟. *Q. E. D.*

Theorem XCIV.

Three only Regular *Figures (that is, whoſe ſides and Angles are =) can fill a ſpace*, viz. *Triangles, as in* A, *Squares, as in* C, *and Hexagons, as in* B.

FOr ſince all the Angles about a point are
[g] *I.* [g] = 4 ∟. 1. The

1. The Angle of a *regular* △ is [h] = $\frac{1}{3}$ 2∟ (becauſe all the 3 Angles are =) and by conſequence 6 of theſe joyn'd together, as in A, are = 4∟ Angles, for $\frac{4}{3}$ 2∟ = 4∟. [h] *X.*

2. The Angle of the □ is [i] ∟; and by conſequence the 4 at C are = 4∟. [i] *Def.*

3. The Angle of a Hexagon is [k] = 1 and $\frac{1}{3}$∟, and by conſequence the 3 joyn'd together in B are = 3 and $\frac{3}{3}$∟, that is, 4∟. [k] *Pre.* 2.

Now all Figures that have more ſides than this, have their Angles bigger than it, and by conſequence 3 of them (which is the [g] leaſt *(number)* that can make a plane Angle) will be ⊏ 4∟; In the Pentagon (of 5 ſides) 3 Angles will be ⌐, 4 ⊏ than [k] 4∟. *Q. E. D.*

Theorem XCV.

Like *Figures*, [S, O, Z, X,] *fram'd a like upon Proportional baſes*, [a . b :: c . d] *will be themſelves alſo Proportional*, viz. S . O :: Z . X.

FOr, $\frac{S}{O}$ [l] = $\left(\frac{A}{B}\ 2 \overset{m}{=} 2\ \frac{C}{D}\right) = \frac{Z}{X}$. *Q. E. D.*

[l] *XCI.*
[m] *LXIII, for A . B :: C . D, Sup.*

Theorem XCVI.

In a Circle, [b a c, b a d,] *the Angles are Proportional to the Arches* [b c, b d,] *that ſubtend them.*

FOr the Line A B, being mov'd towards C, at the ſame time will diſpatch both the Angle

A, and the Arch BC; and by consequence when it comes to the middle of one, it will be at the middle of the other, and so forward; so that in fine, whatsoever part BAD will be of BAC, BD will be the same part of BC: Therefore the Angle BAD, will be to BAC :: Arch B, to BC. *Q.E.D.*

Theorem XCVII.

A Polygon conscrib'd about a Circle is = to a Right Angled Triangle, whose base [ββ] *is the Compass of the Polygon, and its height* [αβ] *the Radius of the Circle.*

DRaw the Radius's AH, AI, &c. to the sides of the Polygon at the point of touching, then take βH = BH; and HC = HC; and so forward, 'till you come to NB; then draw αZ parallel ββ. Erect the Perpendiculars, HO, IP, KQ, &c. and joyn Oβ, OC; PC, PD, &c. Now, because the Angle OHB [n] = (∟ [o] =) αβH, therefore HO is [p] parallel to αβ; and by [q] consequence HO = (αβ [o] =) HA; therefore the △s, BAC and βOC have the same height; therefore BAC [r] = (βOC [s] =) βAC: In like manner CαD = (CPD =) CAD, &c. and, in fine, Gαβ = (GZB =) GAB. So then all the parts being equal, the whole △αββ [t] = to the Polygon. *Q.E.D.*

[n] *Constr.*
[o] *Sup.*
[p] *IV.*
[q] *XXX.*
[r] *XXVIII.*
[s] *XXVI.*
[t] *Ax.* 3.

Theorem

Theorem XCVIII.

Everry regular *Polygon is equal to a Right Angl'd* △, *whose base is the Compass of the Polygon, and its height, a Perpendicular* [a h] *to the side of the Polygon* [b c] *from the Center.*

FOr that Perpendicular *(a h)* will be the same with the Radius of the Circle that may be inscribed in such a Polygon, and than the rest follows from the Precedent.

Theorem XCIX.

A Circle is = *to a* ∟ *Angled* △, *whose base is the Circumference of the Circle, and its height, the Radius of it.*

1. EVery [u] Polygon conscrib'd is ⊏, and inscrib'd is ⊐, then the ○. [u] *Ax.* 5.

2. The compass of a Polygon conscrib'd, is ⊏, and inscrib'd is ⊐, than the [u] circumference of the Circle.

3. This ∟ Angled △ will be [w] ⊐ than any Polygon conscrib'd, and ⊏ than any inscrib'd, (because the circumference of the ○, which is the base of this Triangle is [x] ⊏ than the compass of any inscrib'd.) Therefore it will be = to the ○. [w] *XCVII.* [x] *Ax.* 5.

For if this Triangle be ⊏ than any thing that is bigger than the ○, and ⊐ than any thing that is ⊐ than the ○, it follows that it must be equal to the ○. *Q. E. D.*

Note. And this is call'd the Squaring of a Circle, *viz.* to find a Right-lined figure equal to a Circle; upon this supposition, that the basis given is equal to the circumference of the Circle. But actually to find a Right-line = to the circumf. of a ○, is not yet discover'd Geometrically.

Theorem C.

An equal Legg'd Triangle [abc] *in a Segment, is* ⊏ *than* ½ *the Segment.*

DRaw the Radius FB Perpend. AC, and the Tangent DBE, which will be Parallel to AC, (because the Angle AGB[y] is ∟ [z] = DBG) further draw AD and CE Parallel GB; DC will be a Pgr. and the Triangle ABC half of it; but the whole Pgr. is ⊏ than the Segment ABC; therefore ½ Pgr. *viz.* ABC, ⊏ [a] ½ the Segment. *Q. E. D.*

[y] *Const.*
[z] *XLIX.*
[a] *LXIII.*

Theorem

Theorem CI.

The Triangle DCB *is* ⌐ *than* ½ *the mixt Triangle* ACB.

LEt AB and DC be Tangents: Therefore DC = [b] DA, and Angle (DCF; [c] is ∟ [d] = DCB; therefore the ſubtend. DB [e] ⌐ (DC =) DA: Therefore the Triangle DCB [f] ⌐ (DCA [g] ⌐) ADC; the Triangle mixt, and by conſequence is more than ½ ABC the Triangle mixt. *Q. E. D.*

[b] *L.*
[c] *XLIX.*
[d] *I.*
[e] *XV.*
[f] *LXXVIII*
[g] *Ax.* 5.

From the two Preceding Propoſitions it appears, that figures inſcrib'd and conſcrib'd, if the number of their ſides be doubled, do approach the Circle by more than half the ſpace that was before left. *Note.*

Theorem CII.

A Circle contains more ſpace, than any other figure that is equal to a Circle.

FOr a ◯ is = [h] ∟ Angled △ whoſe baſe is the circumference of the ◯, and height, the Radius AB. But the Square CD (which we ſuppoſe = to the ◯) is [i] = to ∟ Angl'd △, whoſe baſe indeed, is the ſame with the former, (becauſe the compaſs of the Square is ſuppoſed = circumference of the ◯:) but the height is the Perpendicular AE. *Q. E. D.*

[h] *XCIX.*
[i] *XCVIII.*

CHAP. IV. PART I.

Of the Power of Lines.

The Power *of Right-lines, is when they are so plac'd as to comprehend the greatest space; which is when they are set at Right-angles. Therefore a* ▭ *and* □ *are call'd the* Power *of Lines.* Note, *two letters set together* AB *or* AE, *(or with a Note of Multiplication between them,* A×B, A×E,*) signifie a* ▭ *made of those two Lines. And* AA, *or* A×A, *or* Aq, *or* QA, *signifies the Square of the Line* A.

Theorem CIII.

A Rectangle [Z B] *of two whole Lines* [ZB] *is* = *to the Rectangles* [AB+EB] *that are made of one whole* [B] *and of the parts* [A, E] *of the other.*

IF Z = A + E } For to = bases (*viz.* Z, and A+E) there is added an = height (B) [k] therefore the Rectangles are =. *Q. E. D.*

then ZB = AB+EB }

[k] *XXXIV.*

Scholium.

Scholium.

The Rectangle of the whole Lines [Z B] *is also = to the Rectangles of the parts of each Line.* [*viz.* C, D. A, E.] *Fig. 11.*

FOr, $ZB^{l} = \left\{\begin{matrix} AB = \\ + \\ EB = \end{matrix}\right\} \begin{matrix} AC + AD \\ {}^{1} \\ EC + ED \end{matrix}\Big\}^{m}$ therefore, $ZB = AC + AD + EC + ED$. *Q.E.D.*

l *Pre.*
m *Ax.* 6.

Theorem CIV.

A Rectangle [Z A] *of the whole, with one of its parts,* [A] *is = to the* Q *of the same part,* [A] + *to the* □ *of the Parts* [A, E] *together.*

FOr, $ZA^{n} = AA$ (*i.e.* Aq) + Æ. For the same height, A is added to ^{o} = bases Z and A, E: Therefore the □ Z A is = A A + A E. *Q. E. D.*

n *CIII.*
o *Ax.* 5.

Theorem CV.

The Q *of the whole is* = Q *of the parts,* + *to the Rectangles made of the ſame parts, viz.* $Zq = Aq + Eq + 2Æ$.

[p] *CIII.* [q] *Pre.* FOr $ZZ^{p} = \left\{\begin{array}{l} ZE^{q} = \} EE\ (i.e.\ Eq) + EA \} \\ + \\ ZA^{q} = \} AA\ (i.e.\ Aq) + AE \} \end{array}\right.$
that is, $ZZ = Eq + Aq + 2Æ$. *Q. E. D.*

Theorem CVI.

Fig. Preced. $Zq + Eq = 2ZE + Aq$.

[r] *CIII.* [s] *CIV.* FOr $Zq^{r} = \left\{\begin{array}{l} ZE \\ + \\ (ZA^{s} =) Aq + (Æ =) \end{array}\right\} ZE - Eq.$
Therefore $Zq + Eq = ZE + Aq + ZE$. *i. e.* $2ZE + Aq$. *Q. E. D.* In numbers thus, $36 + 4 = 12 + 16 + 12$. Suppoſing Z to be 6, A 4. E 2.

Where *Note*, that a Square, is a number Multiplied by it ſelf; and a Reactangle, two numbers Multiplied by one another. The like inſtance in numbers might be given in the other Propoſitions.

Th-orem

Theorem CVII.

The Q *of the whole* [Z] *and of either of its parts,* [E] *added to it, (as of one line) is* = 4 □s *made of the whole* [Z] *and that same part* [E;] + Q *of the other part* [A].

Viz. Q, Z + E = 4ZE + Aq.
For, Q Z + E = (Z q + E q) + 2 ZE. *Q.E.D.* [t] *CV.*

[u] 2 ZE + Aq [u] *Pre.*

Theorem CVIII.

The Q *of the half,* [½ Z] *is* = □ *of the unequal parts,* [A + E, *and* B] + Q *of the intermediate part* [E].

Viz. Aq (*i.e.* Q, ½ Z) = A + E × B + Eq.
For, A + E × A — E (*i.e.* B) [w] = (Aq — Æ + EA — Eq *i.e.*) Aq — Eq. Therefore, A + E × B + Eq = Aq. *Q.E.D.* [w] *Sch. CIII.*

Theorem CIX.

The Qs of the unequal parts [Z *and* B] *are equal* 2 Qs *of the half* [A] *and of the intermediate part* [E].

Viz. $Zq + Bq = 2Aq + 2Eq$.

For Zq (*i.e.* $QA+E)^{x} = Aq + 2Æ + Eq$.

And, Bq (*i.e.* $Q,A-E)^{x} = Aq - 2Æ + Eq$.

Therefore $Zq + Bq = 2Aq + 2Eq$. *Q.E.D.*

For $+2Æ$ and $-2Æ$ destroy one another.

x CV.

Theorem CX.

The Q of the whole [A] *with the piece added,* [E] *as one line, and of the piece added, is double to the Q of the other half,* $\frac{1}{2}$A, *and of the other half* A *with the piece added,* $\frac{1}{2}A+E$.

Viz. $Zq + Eq = 2Q, \frac{1}{2}A + 2Q, \frac{1}{2}A + E$.

y CV.
z Ax. 5.
a LXXX.

For,

$$Zq + Eq^{y} \left\{ \begin{array}{l} Aq^{z} \left\{ \begin{array}{l} \frac{1}{2}Aq^{a} = \} 2Q\frac{1}{2}A. \\ + \\ \frac{1}{2}Aq^{a} = \} 2Q\frac{1}{2}A \end{array} \right. \\ + \\ 2Æ^{z} = \} 4\frac{1}{2}Æ \\ + \\ 2Eq \end{array} \right\}^{y} = \} 2Q\tfrac{1}{2}A + E.$$

Q. E. D.

Theorem

Theorem CXI.

The Q *of the half with the piece added,* [E,] *is* = *to the* □ *of the whole,* [A] *and the piece added,* + Qs *of the half, and of the piece added.*

Viz. $Q\frac{1}{2}A + E^{b} = \underbrace{\text{Æ}}_{2\frac{1}{2}\text{Æ}} + Q\frac{1}{2}A + Eq.$ [c] ‖ *Q. E. D.*

[b] *C V.*
[c] *Ax.* 5.

CHAP. IV. PART II.

Of the Power of Proportional Lines.

Theorem CXII.

In Proportional Lines [A . B :: C . D] *the* □ *of the extreams* [A D] *is* = □ *of the middle* [BC.]

DRaw out {T F and H E} to G. [d] Becauſe the □ AB is :: CD ([e] ſince A . B :: C . D) [f] therefore they have the ſame Diameter LG: [g] therefore

[d] *Def.*
[e] *Hyp.*
[f] *LXXXIX*
[g] *XXX.*

fore the Triangles $\left\{\begin{matrix} LGH = LGI \\ LKA = LKB \\ KGE = KGF \end{matrix}\right\}$ there-
fore (the = Triangles being omitted) the □
KH = KI. and the common DC being added
[h] *Ax.* 8. to these, DC + KH[h] = DC + KI, that is,
AD = BC. *Q. E. D.*

Scholium.

In ∴ A, B, C. The □ of the extreams is
4, 6, 9. = to the □ of the middle.
viz. AC = Bq. *Note,* This Proposition
36 = 36. is call'd the **Catholic Theorem** and is the Foundation
of the *Golden Rule* in Arithmetick.

Theorem CXIII.

In a Rectangled Triangle [h e i.] *the* Q
of the subtendent [h, i.] *is* = *to th*
2 Q's. *of the* 2 *sides, viz.* A = B + C

FRom the L E, let fall the Pp. E D. Then
[i] *LXXVI.* [i] HI, IE, ID ∴ also IH, HE, HD ∴
[k] *LXXXIII.* [k] Therefore, A. $\left\{\begin{matrix} B \\ C \end{matrix}\right\}$ :: HI. $\left\{\begin{matrix} ID \\ HD \end{matrix}\right\}$ But, [l] HI =
[l] *Ax.* 5.
[m] *Ax.* 10. $\left\{\begin{matrix} ID \\ + \\ HD \end{matrix}\right\}$ [m] Therefore, A = $\left\{\begin{matrix} B \\ + \\ C \end{matrix}\right\}$. *Q. E. D.*

Theore

Theorem CXIV.

In Acute-angled Triangles, the Q. of one side [C,] is less than the Q's of the 2 other [Z, B] (by 2 ZE.)

viz. $Bq + Zq = Cq + 2ZE$. Let fall the (Pp. DE. [n] *Pre.*

For, [n] $\|$

$Dq + Eq + Zq$

[o] $\|$ [o] *CVI.*

$Dq + Aq + 2ZE$

[n] $\|$

$Cq + 2ZE$. *Q. E. D.*

Theorem CXV.

In Obtuse-angled Triangles, the Q. of the subtendent of the Obtuse-angle, is ⊏ than the Qs of the 2 other sides (by 2 Æ.)

DRaw out A, to E, and let fall the Pp. DE. That is, $Bq = Aq + Cq + 2Æ$.

For, [p] II [p] *CXIII.*

$Zq + Dq$

[q] II [q] *CV.*

$Aq + 2Æ + Eq + Dq$

[p] $\|$

$Aq + 2Æ + Cq$. *Q. E. D.*

Theorem

Theorem CXVI.

In Triangles, that Angle is right, th
Q. of the whose ſubtendent is = to th
Q's of the other ſides.

FOr if the Angle H E I, were Acute or Ob
tuſe, the Q. of the ſubtendent, H I, woul
[r] *CXIV.* be [r] ⌝ or [s] ⌜, then the Qs. of the other ſide
[s] *CXV.* contrary to the ſuppoſition: Therefore it is ⌞
Q. E. D.

Theorem CXVII.

All like Figures made upon the ſubtende
of an ⌞, *are = to thoſe made upon th*
other 2 ſides, viz. A=B+C.

FOr, A, B, C, are to one another as the Q
[t] *XCI.* [t] of their Homologous ſides, D E, D F, F
[u] *CV.* But the Q. of D E is = [u] Q, D F+Q, F
[w] *Ax.* 10. [w] Therefore the Figure A=B+C. *Q. E. D*

Theore

Theorem CXVIII.

The ſame thing is true of Semicircles.

Or ○s are to one another as the [x] Q's of their Diameters, [y] therefore alſo Semicir-es. But the Q. of the Diam. F H is = to both e Qs. of the Diam's. F G, G H, therefore the emicircle B A = B D + C E. *Q. E. D.*

[x] *XCII.*
[y] *LXIII.*

Thorem CXIX.

he Squaring of the Lunes (or, half moons) of Hippocrates of Scio.

B Ecauſe the Semicircle A B [z] = B D + C E; therefore, leaving out the common B and , there remains the △, A = to the 2 Lunes, + E.

F. CXVIII.
[z] *Pre.*

2. When the △ is = l egg'd, the Lunes re equal, (the Diameters of the Semicircles eing equal) and both together (D + E) are =[a] to the whole △, I K: Therefore either of hem are = to ½ the △, *viz.* I or K.

Fig. CIX.
[a] *CXIX. 1.*

3. But when the △ is ſcalene, it is as difficult o divide it, by the Line G Z, into 2 parts = o the reſpective Lunes D, E; as to find the quare of a Circle. Of which we ſpoke before n the Note on Theorem XCIX.

F. CXVIII.

Theorem

Theorem CXX.

1. *Lines in a Circle*, [a b, d c,] *cut themſelves proportionally.* 2. *And their parts make* = □s, *viz.* □BEA=CED.

1. FOr the Angle $\left\{\begin{matrix} A = D \\ C^{b} = B \\ DEB^{c} = AEC. \end{matrix}\right\}$ [d] Therefore the △ DBE is like AEC; and by [e] conſequence BE . CE :: ED . EA. 2. Therefore the □ BEA[f] = CED. *Q. E. D.*

[b] *LIV.* [c] *III.* [d] *LXXI.* [e] *Def.* like. [f] *CXII.*

Theorem CXXI.

Lines between Paralls. cut themſelves Proportionally, *viz.* A . B :: C . D.

FOr the △ X, is [g] like Z, (becauſe the Angle I = [h] H, and G [h] = K, and the two head Angles are [i] =) and by conſequence:

E . A :: F . B
E . C :: F . D } therefore alterned, E, F :: { A . B / C . D

[k] Therefore A . B :: C . D. *Q. E. D.*

[g] *LXXI.* [h] *IV.* [i] *III.* [k] *Ax.* 13.

Theorem

Theorem CXXII.

The □ *of the whole Secant with the part added, is* = *to the* Q. *of the Tangent, drawn from the end of the Secant.*

Viz. □ CAE = Q, AD. For the Angle {ADE[l] = ACD, A, is common:} [m] Therefore AED = CDE: [n] Therefore the △ CAD is like AED; therefore, AC. AD. AE ∴ and by [o] conſequence the □ CAE = Q, AD. *Q. E. D.*

[l] *LX.*
[m] *XXII.*
[n] *LXXI.*
[o] *CXII.*

Theorem CXXIII.

Lines drawn from a point without, to the inward Circumf. of a O, *are to one another, as their parts without the* O. *viz.* A+B. C+D :: D. B.

For the △ HEK is [p] like IGK; ([q] becauſe the Angle K is common, and H is [r] = G;) therefore A + B. D :: C + D. B; and *Alterned*, A + B. C + D :: D. B. *Q. E. D.*

[p] *LXXI.*
[q] *XXII.*
[r] *LXX.*

Theorem

Theorem CXXIV.

If the Diameter of a ○ [a c] *be cut by infinite Perpendiculars, either in, at, or without the* ○, *and a Secant* [a e] *be drawn, it will be,* A D . A C :: A B . A E.

[r] *LXXI.* [t] *XXII.* [u] *Hyp.* [w] *LVI.*

BEcauſe the △s E A C and D A B are [r] like; ([t] for the Angle E C A [u] = ∟ [w] = A D B; and A is common,) therefore A E . A C :: A B . A D, and *Alterned Inverted,* A D . A C :: A B . A E. *Q. E. D.*

Scholium.

IN the ſecond caſe (*viz.* at the ○) A C, being the ſame with A B, A D . A B . A E are ∴; and by conſequence the Diameter [A B] is a middle Proportional between the whole ſecant [A E,] and the internal part of it [A D].

Theorem CXXV.

A Chord [a d] *dividing equally an Angle* [b a c] *in a Segment is (it ſelf, and its leſſer part) reciprocally proportional to the ſides of the Angle, viz.* A C . A E :: A D . A B.

[x] *LXXI.* [y] *XXII.* [z] *Hyp.* [a] *LVI.* [b] *Def.* 54.

BEcauſe the △s, A B D and A E C, are [x] like; (for [y] the Angles at A are [z] =, and D [a] = C;) [b] therefore A C . A E :: A D . A B. *Q. E. D.*

Theorem

Theorem CXXVI.

The Q. Circumscrib'd, (or the Q. of the Diameter) is double to the Q. inscrib'd, in a ○, viz. EFq = 2ECq.

FOr, EFq (*i.e.* AB) = [c] CFq + CEq (the Angle ECF, [d] being ∟) and EC [e] = CF: Therefore EFq is double to either of them, CFq, or CEq, *i.e.* CD. *Q. E. D.*

[c] *CXIII.*
[d] *LVI.*
[e] *Def. of Q.*

Theorem CXXVII.

A Trapezium (that is, an irregular *four sided figure) being inscrib'd in a Circle: The* □ *of the diagonals* [ACDB] *is = to the* 2 □'s *of the opposite sides; viz.* DC×AB, DA×CB.

TAke the ∠ADE = BDC. Therefore the △s DCE and DAB, also DAE and DBC are [f] :: (For ∠ { DCA [g] = DBA. / ADB [h] = EDC. } ∠EDB being common, and ADE being [i] = BDC: Also ∠ { DAE (that is DAC) [i] = DBC. / ADE [h] = BDC. })

[f] *LXXI. & XXII.*
[g] *Ax.* 8.
[h] *Const.*
[i] *LIV.*

[k] *Def.* 45. And by [k] consequence $\begin{cases} DC.CE :: DB.BA. \\ DA.AE :: DB.BC. \end{cases}$

[l] *CXII.* Therefore [l] $\square \begin{cases} DC, BA = CE, DB. \\ BA, BC = AE, DB. \end{cases}$

[m] *CIII.* Therefore $\square$ DC, BA + DA, BC = (DB, CE + DB, AE [m] =) DB, AC. *Q. E. D.*

CHAP. V.

Of Surfaces.

Theorem CXXVIII.

No part of a Right-Line [b c] *can be out of a Plane, when any part of it, is in it: That is, no Right-Line can lye upon two Planes.*

FOr if you say, that ABC, is part in the Plane, and part out, then by drawing the Line AC, AC will be [n] shorter than ABC. Contrary to *Def.* 4.

[n] *XI.*

Theorem

Theorem CXXIX.

1. *Every Right-lin'd* △ *is in the ſame Plane.*
2. *And alſo* 2 *Right Lines croſſing one another.*

1. FOr a [o] △ is a Plane Surface, comprehended within three Lines. [o] *Def.* 13.

2. If therefore, AB, CB are in the ſame Plane, AE, CD, [p] will be alſo in the ſame Plane. [p] *Pre.*

Theorem CXXX.

Parallel Right-Lines [ab, cd] *are in the ſame Plane.*

DRaw the Right-Lines AD, CB. The [q] △ CED is in one Plane; and alſo the △ AEB: But DE and EA are in the ſame Plane: Therefore both the △s are in the ſame Plane; and by conſequence their baſes, AB, CD. *Q. E. D.* [q] *Pre.* 1. [r] *Pre.* 2.

Theorem CXXXI.

The interſection [a b] *of two Planes, is a Right-Line.*

IF A B, be not right, draw the right A G B in the Plane C D; and A H B in the Plane E F; that is, two Right-Lines between the ſame points. Contrary to *Def.* 4.

Theorem CXXXII.

The interſections [a b, C D] *of two Parallel Planes, by a third Plane, are Parallel Lines.*

IF not, let A B, C D, meet in I: Therefore the Planes E F, G H, ſhall meet there alſo, [CXXVIII] (becauſe A B I, and C D I are in the ſame Planes) Contrary to *Def.* 65.

Theorem

Theorem CXXXIII.

A Line [dc] *that is Pp. to two Lines* [t] *crossing*, [ac, bc] *is also Pp.* [u] *to their Plane* [af.]

[t] *CXXIX.* 2
[u] *Def.* 50.

IF not, let DE be Pp. to the Plane AF, and from the point E, draw the Right-Lines EA, EC, to which, the said DE ought to be Pp. by *Def.* 60.

Now, $\square AD^{a} = \begin{cases} AC^{b} = \begin{cases} AE \\ + \\ EC \end{cases} \\ + \\ DC^{b} = \begin{cases} ED \\ + \\ EC \end{cases} \end{cases}$

[a CXIII. because the Angle ACD, is ∟ by Hyp. b because the Angles AEC, dec, ought to be ∟ by *Def.* 60.]

Therefore the $\square$, AD $\sqsubset$ Qs. AE + ED: Therefore the Angle DEA, cannot be ∟. [CXIII.] (the same reason will hold, if the Line DE, touches the Plane any where but in C) and by consequence not DE, but DC [w] is Pp. to the Plane. *Q. E. D.*

[w] *CXVI.*

Theorem CXXXIV.

Three Lines which receive the ſame Pp. [E A] *are in the ſame Plane.*

IF not, let A B, be in another Plane [*viz.* E F] cutting the Plane G H, in A F: Since therefore, E A is Pp. [x] to A C, A D, it will alſo be Pp. to their Plane, G H: Therefore the Angle E A F, [y] is ∟, as is alſo [x] E A B, and by conſequence E A F is [z] = E A B; that is, the part = to the whole. *Q. E. A.*

[x] *Hyp.*
[y] *Def.* 60.
[z] *Ax.* 2.

Theorem CXXXV.

If of two Parallels, one [a b] *be Pp. to a Plane, the other* [c d] *is alſo.*

IF not, let E D be Pp. This will be in the Plane [a] D F. Now, the Angle E D B [b] is (∟ [c] = A B I) = [d] C D B, *viz.* part = whole. *Q. E. A.*

[a] *CXXIX.*
[b] *Def.* 60.
[c] *Ax.* 2. *and Hyp.*
[d] *IV.*

Theorem

Theorem CXXXVI.

One only Pp. [cd] *can be rais'd from one point in a Plane.*

IF it is poſſible, let CE be alſo Pp. and through theſe two CE, CD, let a Plane be [e] drawn; cutting the other Plane in ACB, the Angles ACE, ACD, muſt be both [f] ⌊; and by conſequence =, *viz.* part = whole. *Q. E. A.*

[e] *CXXIX.*
[f] *Def.* 60.

Theorem CXXXVII.

A Pp. [fe] *to one Plane,* [ab] *is Pp. to its Parallel Plane.*

DRaw a Plane through FLE, cutting the others in LH and EG, theſe interſections will be [g] Parallel. Now the Angle GEF, is ⌊ [h], and = [i] HLF: (And the like, if the Plane paſs through EM, LO,) [k] therefore EF is Pp. to the Plane, CD. *Q. E. D.*

[g] *CXXXII.*
[h] *Hyp.* [i] *V.*
[k] *Def.* 60.

Theorem CXXXVIII.

Planes are Parallel that receive the ſame Pp.

[l] *XVI.* FOr ſince the Angle GEF = HLF, [l] therefore the Line HL is Parallel EG, (and the like of ME to OL, *&c.*) whereſoever the interſection can happen; therefore the Plane AB is Parallel to CD. *Q. E. D.*

Theorem CXXXIX.

If two croſſing Lines, [b a c] *are Parallel to two croſſing Lines* [e d f] *in another Plane, their Planes alſo will be Parallel.*

[m] *Hyp.*
[n] *Def.* 60.
[o] *VI.*
[p] *Conſtr.*
[q] *CXXXIII*
[r] *Pre.*

DRaw AG Pp. to the Plane EF, and HGI, Parallel (EDF, [m] Parallel) BAC: Therefore the Angles HGA, IGA [n] are ∟ [o] = CAG, BAG: Therefore AG is Pp. to the Plane [p] EF and [q] BC: And by [r] conſequence the Planes BC, EF are Parallels.

Theorem

Theorem CXL.

Two meeting Lines [b a c] *Parallel to two* [e d f] *in another Plane, contain an = Ang. with them,* viz. *Ang.* A = D.

TAke AB = DE, and AC = DF, and joyn BE, AD, CF: Since AB, ED, and AC, DF, are [f] Parallel and [t] =; BE is also [u] = and Parallel, (to AD [u] = and Parallel) [w] CF. Therefore BC [u] = EF; and by consequence the △BAC [x] = EDF, and the Angle A = D. *Q. E. D.*

[f] *Hyp.* [t] *Constr.* [u] *XXX.* [w] *VIII.* [x] *XVI.*

Theorem CXLI.

Parallel Lines [a e, b f] *are cut proportionally by Parallel Planes.*

Viz. AC . CE :: BD . DF: Because the intersections AB, CD, EF, are [y] parallels; [z] therefore $\frac{BD}{DF} = \left(\frac{BZ}{ZE} =\right) \frac{AC}{CE}$. *Q. E. D.*

[y] *CXXXII* [z] *LXVII.*

PART. II.

Of a Solid Angle.

Theorem CXLII.

Of three Angles which are neceſſary to compoſe a Solid Angle [a] *any two, are* ⊏ *than the third.*

IF three Angles are =, it is evident: But if one be biggeſt, *viz.* BAC, take BAE = BAD; joyn BC, and by the Lines BD, CD, take off AD = BE: Becauſe, AD = AE, and AB is common, and the Angle BAE [a] = BAD: [b] Therefore BD = BE; but BD + DC [c] ⊏ BC: Therefore (BD being [a] = BE) DC [d] ⊏ EC; and by conſequence the Angle DAC [e] ⊏ EAC; therefore (BAD being [a] = BAE) BAD + BAC [d] ⊏ BAE + EAC, *i. e.* BAC. *Q. E. D*

[a] *conſtr.*
[b] *XVII.*
[c] *XI.*
[d] *Ax. 9.*
[e] *XIX.*

Theorem CXLIII.

A Solid Angle [a] *is made up of leſs than Four* ∟ *Angles.*

FOr, the 6 Angles at B, C, and D, + to the 3 at A, are [f] = 6 ∟; but the 6, at B, C,

[f] *X.*

C, and D, are [g] ⊏ than (the 3 B, C, D, in the base of the Pyramid [f] =) 2 ∟ : Therefore the 3 at A, are ⊐ 4 ∟. *Q. E. D.*

[g] *Pre.*

The reason is this, since the Four Angles made by 2 Lines crossing, (as at Z) are [h] = 4 ∟ ; The Plane [i] in which they are cannot be so folded (in the Lines Z F, Z G, Z E, Z H) as to make a Solid Angle at Z, unless something be left out from one of the Angles, as F Z S.

[h] *I.*

[i] *CXXIX.*

CHAP. VI.

Of Bodies, or Solids.

Theorem CXLIV.

The opposite Planes [a c, d b] *of a Parallelepipedon* (*Ppp.*) *are like and equal.*

A C, D B, are [k] Parallel: Therefore the intersections A F, D E are [l] Parallel; and A F D E is a [m] Pgrm. by consequence A F [n] = D E. After the same manner it may be prov'd that all the other opposite Lines are Parallel and =: Therefore the Angle F A H = E D G, and A F C = D E B. Lastly, the Plane A C is [o] = and [p] like D B, and so of the rest. *Q. E. D.*

[k] *Def.*

[l] *CXXXII.*

[m] *Def.* 20.

[n] *XXX.*

[o] *XXXIV.*

[p] *Def.*

Theorem

Theorem CXLV.

A Ppp. is divided in the middle, by a Plane [a h] *that passes through the Diameters* [e a, h m.] *of two opposite Planes.*

ALL the opposite Pgrs. are [q] =: The Δ AED [r] = AEF, and LMH [r] = MHG, the Plane AH is common, therefore the Part or Prism AL = AG. *Q. E. D.*

[q] *Pre.*
[r] *XXX.*

Theorem CXLVI.

Ppps. [ZX *and* ZS] *are* = *which have the same base and height.*

(*Note*, This Figure would be best comprehended, if it were cut out in Cork, or the like matter.)

1. LEt them fall between the same Parallels, KF, and DA: Therefore the Prism (EDH, *i. e.*) S + O = O + X. [And (the common O, being left out) S = X, and (Z being added) S + Z = X + Z. *Q. E. D.*]

Because, the Plane 1. EPH [t] = FRA, EH being [t] = FA, EP [t] = FR Ang. P = [u] R, 2. The Planes opposite to these, are = by the same reason. 3. DE [w] = GF, and LE [y] = CF, being between the same [x] Parallels, and the bases KE [t] = MF. 4. GA = DH; because DR [w] = (KF [w] =) LA: Therefore DR + GH [z] = LA + GH. [a] Therefore S + O = O + X. 2. If

[t] *XXIV.*
[t] *XXX.*
[u] *IV.*
[w] *CXLIV.*
[x] *Hyp.*
[y] *XXXIV.*
[z] *Ax.* 8.
[a] *Def.*

2. If a Ppp. of the ſame baſe and height with Z X, ſhould not fall between the ſame Parallels; It will however fall between the ſame Paralſels with S Z; and being, as before, = prov'd to it, will be [b] = to Z X. [b] *Ax.* 6.

Theorem CXLVII.

Ppps. [S Z, O] *are* = *which have an* = *and like baſe, and the ſame height.*

UPon the baſe C A B, ſuppoſe a Ppp. fram'd Z X, of the ſame height, and like to O. Then, O = Z X. (for the ſeveral ſides will be prov'd = by XXXIV. and *Like*, by conſtruction) [c] *Pre.* which Z X [c] = S Z; therefore O [d] = S Z. [d] *Ax.* 6. *Q. E. D.*

Theorem CXLVIII.

Ppps. [a b, a d.] *of the ſame height, are to one another as their baſes.*

LEt the baſe C G, be made = C E; therefore the Ppp. S = A H: Now Let the Plane B F, (being [e] Parallel G H and A C) be mov'd Parallely from A C to G H. It ſhall at the ſame time diſpatch the baſe G C, and the Ppp. A H: And by conſequence how much ſoever it has taken away at any time from the baſe, it will have taken away a *Like* part from the Ppp. ſo that, as the baſe G C is (for inſtance) to its part G F:: Ppp. A H (= S X.) *Q. E. D.* [e] *Def. Ppp.*

Theorem

Theorem CXLIX.

Equal Ppp's. [S = XO] *have their baſes and height, reciprocally proportional,* Def. 46. *viz.* AB . CD :: MK . AI.

TAke the Ppp. X, of the ſame height as S, that is, ML = AI. Then, $\frac{AB}{CD}$ [f] = $\left(\frac{S}{X}\right)$ [because O + X = S. Hypo.] = $\frac{O+X}{X}$ [f] = $\frac{DK}{DL}$ [h] = $\frac{MK}{ML}$; that [e] is $\left.\right)\frac{MK}{AI}$ (becauſe ML = AI, Conſtr.) *Q. E. D.*

[f] *Pre.* [e] *Ax.* 10. [h] *LXXX.*

Theorem CL.

Thoſe Ppps. [S = XO] *are equal, whoſe baſes and heights are* [a] *reciprocally proportional, viz.* AE . FC :: DN . BE.

[a] *Def.* 46.

FOr, Ppp. $\frac{S}{X}$ [i] = $\left($*baſ.* $\frac{AE}{FC}$ [k] = *height* $\frac{DN}{BE}$, *i.e.* MN = *ſide* $\frac{DF}{HF}$ [m] = *Pgr.* $\frac{DK}{HK}$ [i] = $\left.\right)\frac{XO}{X}$. Therefore S [n] = XO. *Q. E. D.*

[i] *CXLVIII.* [k] *Sup.* [l] *LXVII. with LXI.* 3 [m] *LXXX.* [n] *Ax.* 10.

Theorem

Theorem CLI.

Like Ppps. [S :: O] *are in a triplicate reaſon of their homologous ſides, viz.* S . O :: AE . ED. *thrice.*

BEcauſe the $\angle DEF^{o} = AEH$, therefore [p]AE, ED may be plac'd in a Right Line, fill up the void ſpace with X and Z: Now,

$$\left.\begin{array}{l} \frac{S}{X}{}^{q} = \left(\frac{AH}{EI}{}^{r} =\right)\frac{AE}{ED} \\ \frac{Z}{X} = \frac{LE}{IN}{}^{r} \\ \frac{O}{X} = \frac{HE}{EF}{}^{r} \end{array}\right\} \text{Alſo, } \frac{AE}{ED} = \frac{LE}{IN} = \frac{HE}{EF}.$$

(For $\frac{AE}{LE}{}^{f} = \frac{ED}{(FC, i.e.)IN}$ and *Alterned* $\frac{AE}{ED} = \frac{LE}{IN}$.

Again, $\frac{LE}{HE} = \frac{(FC\ i.e.)IN}{EF}$, and *Alterned* $\frac{LE}{IN} = \frac{HE}{EF}$) So then it appears that, $\frac{S}{X} = \left(\frac{AE}{ED} = \frac{LE}{IN} =\right)\frac{X}{Z}$ $\left(\frac{LE}{IN} = \frac{HE}{EF} =\right)\frac{Z}{O}$: Wherefore $\frac{S}{X} = \frac{X}{Z}{}^{t} = \frac{Z}{O}$; that is, S, X, Z, O $\div\div$ and by conſequence $\frac{S}{O} = \left(\frac{S}{X}\right.$ thrice, which $\frac{S}{X} =\left.\right) \frac{AE}{ED}$ thrice,

[o] *Def.* 45. [p] *III.* [q] *CXLVIII* [r] *LXXX.* [f] *Def.* 45. [t] *Ax.* 10. [u] *Def.* 42.

E. D.

Scholium,

Scholium to this, and the former Theo.

BEcauſe Ppps. are extended in length, breadth and depth; therefore that the reaſon of one Ppp. to another, may be known, we are to take the reaſons of all the dimenſions together, viz. *if a Ppp.* A, *be twice as broad, three times as long, and four times as high as* B; *then* A *is twice* 3 *times* (*i. e.* ſix times) 4 *times* (*i. e.* 24 times) *as great as* B. *But if the Ppps. are* like: *Then the reaſons of their length, breadth and depth are the ſame*[a]: *So that if* A *be twice longer, it muſt be alſo twice broader and twice higher than* B. *That is, twice twice* (*i. e.* 4 times) *twice* (*i. e.* 8 times) *greater than* B. *So that the ſame reaſon is taken* 3 *times; and is thereupon call'd* Triplicate Reaſon.

[a] *Def.* 45.

Theorem

Theorem CLII.

Like Ppps. are to be one another, as the Cubes of their Homologous *ſides.*

BEcauſe all Cubes are *like* Ppps. and by conſequence are to one another in a Triplicate Reaſon of their ſides (by Prec. Theor.) But like Ppps. are in the ſame [a] reaſon. Therefore they are to one another as the Cubes rais'd upon their Homologous ſides. *Q. E. D.*

[a] *CLI.*

Theorem CLIII.

WHat has been ſaid in the foregoing Propoſitions concerning Ppps. does agree alſo to Priſms. (as being the [a] halves of a Pp) Provided that their oppoſite Parall. Planes, if they are not △s, be reſolv'd into △s.

[a] *CXLV.*

Theorem CLIV.

They agree alſo to Cylinders. For Example. (according to CXLVI) *If Cylinders have the ſame baſe* [G B] *and equal height then they are equal.* viz. AB=BC.

IF not; let AB be ⌐; and let there be inſcribed in it a Priſm of many ſides (as ADFE) ⌐ alſo then the Cyl. BC. (Becauſe by multiplying the ſides of the Priſm, you may approach by infinite degrees nearer and nearer to the Cyl. in which it is inſcrib'd. Beſides that, though the degree by which the Cyl. BC. is leſs than BA, be ſuppoſed to be never ſo ſmall, yet it is ſtated and fixt, and may not (after it is given) be alter'd. Whereas our Approaches in the Priſm may be made to Infinity.) Then upon the ſame baſe, GF, let there be inſcrib'd a Priſm in the Cyl. BC; Theſe Priſms will be [a] equal, that is, the Priſm HCF (=DAF) ⌐ [b] Cyl. BC. A part greater then the whole. *Q.E.A.*

[a] *Pre. & CXLVI.*
[b] *Conſtr.*

Theorem

Theorem CLV.

A Plane [l m n] *cutting a Pyramid* [b d] *Parall. to the baſe* [a b c] *makes a figure like the whole.*

FOr the Ang's at the top are common, and thoſe at the baſes, may be proved = by Theor. IV. Then, that the ſides about theſe reſpective ∠s are proportional, appears thus; A B . L N : : [a] A D . L D; and alterned; A B . A D : : L N . L D and ſo of the others. Wherefore the figures are [b] like. *Q. E. D.*

[a] *LXIX.*

[b] *Def.* 45.

Theorem CLVI.

The Sections [l m n o p r] *of two Pyramids* (*whoſe baſes, and heights are* =) *made by a plane* [l n r p] *Parall. to both their baſes, are equal.*

BEcauſe the baſes, A B C, E F G are : : [a] the Sections L M N, O P R; therefore $\frac{ABC}{LMN}$ and $\frac{EFG}{OPR}$ in a duplicate [b] reaſon of their Homologous ſides, but A B C is = [c] E F G. Therefore, L M N = [d] O P R. *Q. E. D.*

[a] *Prec.*

[b] *XCI.*

[c] *Sup.*

[d] *Ax.* 10.

Theorem CLVII.

All Pyramids are equal, which have equal baſes and height.

FOr inſtance C D = F H. If not, let C D be ⊏; and inſcribe in this a ſolid Segment, (made up of Priſms having *like* baſes and = height, L N M) ⊏ than the Pyram F H. (In which we argue, as before we did in Th. CLVII. becauſe, the height of the Priſms being leſſen'd, they may be infinitely multiplied, ſtill approaching toward the Pyram. in which they are inſcrib'd; till at laſt they may be made to come nearer to it than any other quantity that has been already given, as F H.) Then let there be inſcrib'd alſo in F H, as many Priſms (for there can be no bound to the number.) Both theſe Segments of Priſms ſhall be =; (for their number is =, *by Conſtruction*; Their baſes and heights may be equal, becauſe thoſe of the Pyramids in which they are inſcrib'd, are = *by Suppoſition)* that is, the Prz. in H F (= Prz. in D C) ⊏ [a] Pyram. H. F. *Q. E. D.*

[a] *Conſtr.*

Theorem

Theorem CLVIII.

Every triangular Prism *(that is, which has a* △ *for its base) may be divided into* 3 *equal Pyramids*; viz. EZD,A. ABC,D. AFC,D.

FOr the base ABC = [a] EFD; and the height is equal. [a]Therefore, ABC,D = [b] EFD,A = AFC,D. Because the base AEF = [c], [a] AFC. and the height at D is common. *Q. E. D.*

Note, This Figure will be plain, if cut out in Cork.

[a] *Def.* 61.
[b] *Prec.*
[c] *XXX.*

Theorem CLIX.

WHat has been demonstrated of *Prisms*, is true also of *Pyramids*, as being the third part of a *Prism*. [a]

[a] *LXIII. & Prec.*

Theorem CLX.

What has been demonstrated of Pyramids *(in the Precedent Theor.) agrees also to* Cones.

FOr instance. The Cone ABC = DEF; having = bases and height. If not, let

A B C, be ⊏; and let there be inſcribed in this a Pyram. A G H C, ⊏ Cone D E F. (which, that it may be done, we prove by multiplying the ſides, as we have done before,) Then (becauſe the baſes of the Cones are = [a]) let there be inſcrib'd in the Cone D E F, a Pyram. E D K L M = [b] and :: A G H C. Now, the Pyram. E D K L = (Pyr. A G H C) ⊏ [c] cone E D K. that is, a part ⊏ whole. *Q. E. A.*

[a] *Sup.*
[b] *Prec.* being the ſame height.
[c] *Conſtr.*

Theorem CLXI.

All like *Bodies are in a Triplicate Reaſon of their Homologous ſides.*

BEcauſe they may be reſolv'd into Triangular Pyramids, equal in number, [a] and like. [a] But ſuch △r Pyramis: are in a [b] Triplicate Reaſon of their homol. ſides; therefore [c] alſo the Bodies. *Q. E. D.*

[a] *Def.* 45.
[b] *CLIX*
CLIX.
[c] *LXIII.*

Theorem CLXII.

A Sphere *is* = *a* Cone *whoſe perp. axis is the radius of the* Sphere, *and its baſe* = *the whole ſurface of the* Sphere.

THe ſame Demonſtration ſerves here as in Theor. XCVII. by ſhewing that all Polygons circum-

circumscrib'd, or inscrib'd in a *Sphere* are ⊏ or ⊐ than such a *Cone*. Therefore the *Sphere* is = to such a *Cone*. *Q. E. D.*

Theorem CLXIII.

Of all solid figures (having an = surface) The Sphere *is the greatest.*

THis also is demonstrated from the Precedent, by Theor. CII.

Theorem CLXIV.

Spheres are in a Triplicate Reason of their Diameters; *or, as the Cubes of their Diameter's* (*Theor.* CLI. CLII.) viz. *Sph.* $\frac{ABC}{DEF} = \frac{AC}{DF}$ *thrice.*

IF not, let the Sphere $\frac{ABC}{DEF}$ be ⊏ $\frac{AC}{DF}$ thrice; and let there be inscrib'd in the Sph. A B C a solid figure of many sides, A B C G, ⊏ to the side D E F than A C to D F *thrice.* (since by continual doubling [a] the sides of the body inscrib'd, as

[a] *Theor.* C.

as AI, IB, &c. whereby the same line AC will always continue to be both the Diam. of the ○, and the side or half the Polygon, we may approach still nearer to the Sph. ABC, and by consequence, make it bigger than any other body, which is less than the Sphere.) Then let there be inscrib'd in the other Sphere a *like* body of many sides. These bodies, being resolv'd into Pyramids and so into △s. [b] will be in a Triplicate Reason of their homol. sides, AC, DF. *viz.* the [c] Body ABCG. DEFH :: AC. DF *thrice.* But $\frac{ABCG}{DEF} \sqsubset^{d} \frac{AC}{DF}$ *thrice.* Therefore the Sph. DEF, is ⊐ than the Body inscrib'd in it, DEFH. *Q. E. A.*

[b] *CLXI.*
[c] *Ax.* 14.
[d] *Constr.*
[e] *Ax.* 11.

Theorem CLXV.

There can be but 5 **regular** *Bodies, (which have all their sides, and all their Angles* =)

FOr no plane figures, joyn'd together, can make a solid Angle; except a △, □ and *Pentagon.* For a solid Ang. ought to consist of less [a] than 3 ∟s. and 3 plane ∠s are the fewest that can make up a solid one (as is naturally evident.) Now 3 ∠s of a Hexagon are = 4 ∟; and in all figures of more sides than a Hexicon. they exceed 4 ∟. Wherefore 3, 4, and 5 △'s

[a] *XLIII.*

△'s (for 6, make [b] 4 ∟s) 3 □, and 3 *Pentagons* (which make [c] $\frac{2}{10}$ of 4 ∟s) may all compose a solid Angle. Upon this account there can be but 5 *regular* Bodies. *Viz.*

[b] *XCIV*, 3.
[c] *XCIV.*

1. *A Tetraëdrum.* [d]	Whose Angle is made up of	3, 4, 5	Equilateral △s.
2. *An Octaëdrum.*			
3. *An Icosaëdrum.*			
4. *A Hexaedrum.*		4	□s
5. *A Dodecaëdrum.*		3	*Pentagons.*

[d] *Def.* 74. *&c. Fig. CLXIV.*

If these, or *like* figures, be cut in Past-board according to the figures in the Type, and folded up, they will represent the foresaid 5 regular Bodies.

Theorem CLXVI.

It will not be amiss in conclusion of the Theorems, to add one Proposition that may serve as an Introduction to the Doctrine of Infinites. *viz. That* Infinites *may be* actually *number'd or measur'd.*

IF from the line A B you take (*suppose*) a fourth part toward A; [A C] and again towards B, 2 such parts, [D B] (*viz.* such a number of parts, less by 2, than the whole line was first supposed divided into) there will remain C D, one fourth of A B. If again, from this *remainder,*

der, you take as before, one part towards A, [C E] and 2 ſuch towards B, [F D] there will remain only E F one fourth of C D. And ſo if you continue to do to every remainder, there will always remain between the lines laſt taken, one fourth of the line from whence they were taken. From which fourth part, there may ſtill after the ſame manner be ſuppoſed to be taken two other ſuch lines on each ſide; but if this be done infinite times *actualy*, then there will nothing more remain (between) and ſo the continu'd diviſion on either ſide will come exactly to the point G; which cuts off a third part of the line A B. (*viz.* a part, of one Denomination leſs, than the whole line is firſt divided into.) Becauſe there was always taken away twice as much towards B as towards A. The total ſum, therefore, of all together that is taken away towards B, will be twice as much as the Total Sum towards A. And by conſequence, the *meeting* will be at ſuch a point [G] as cuts off G B double to G A. Which was the thing undertaken; viz. *to aſſign the preciſe meaſure of a line infinitely divided.*

But if it be inquir'd how this diviſion can be made infinite times *actually*; I anſwer; Let two points begin to move from A and B, at the ſame time; That at B, always moving as faſt again as that at A. It is certain they will at length meet; and (by the former Demonſtration) juſt at the point G. For the point B, will be at D, when A is at C. and B again at F, when A is at E. and ſo they will each paſs through the infinite Sub-diviſions before mentioned; and when they meet, will have divided the line A B infinite times *actually*. The

The ſame may be ſaid, if one of the points be to overtake the other; as in the famous Sophiſm of *Zeno*, who argu'd that a Horſe would never overtake a Snail. For ſuppoſe the Horſe run ten times as faſt as the Snail, and the Snail were a Mile before, and both ſet out at the ſame time. When the Horſe has run the firſt Mile, the Snail will have got to the 10th part of the 2d Mile; when the Horſe has run this 10th part, the Snail will be got to the 10th of the next 10th part; (that is, a 100th part of the 2d Mile) and when the Horſe has got this, the Snail will be a 1000th part before him. And ſo in infinite Subdiviſions, when the Horſe has got the laſt part, the Snail will ſtill have got a part before him. On the contrary, it is naturally evident, as any axiom, that the Horſe will at length overtake the Snail; and (by conſequence) meaſure out thoſe infinite Sub-diviſions *actually*; The ſuppoſion of the Impoſſibility of which, is the ground of the Fallacy. Now the preciſe point of meeting may be determin'd, by a very eaſy Demonſtration. *viz. at the end of the 9th part of the 2d. Mile.* For ſince the Horſe runs 10 times as faſt as the Snail, the Horſe will run $\frac{10}{9}$ of a Mile, while the Snail runs $\frac{1}{9}$; and by conſequence they will come both at the ſame time to the end of the firſt 9th part of the 2d Mile. *Q. E. D.*

Theorem

PROBLEMS

Which purpose something to be done.
Demands, or Suppositions.

1. *That a right line may be drawn from any one point, to another.*
2. *That a right line may be continued at either end as far as we please.*
3. *That a Circle may be describ'd upon any Center, and at any distance, (or interval.)*

Problem I.

From a point given [a] *to draw a right line* [a e] *parall. to another* [b c].

ON the Cent. A, at any distance, (so as to cut the line B C) describe the Arch, D G E, Cent. D, same distance describe another Arch cutting the line given (B C) in G. Lastly, Cent. G same distance, cut the first Arch in E, the line

e AE is parall. BC. For AD = [a] AE = [b] G = [b] GE, and AG is common. There- re in the △'s AEG and AGD, the ∠ GAE = [c] AGD. Therefore [d] AE is parall. to BC. *Q. E. F.) which was to be done.*

[a] *Radius's of the same* ○.
[b] *Rad's of* = ○s.
[c] *XIII.*
[d] *VI.*

Problem II.

rom a point given, [a] *to draw a line*, [a e] *equal to a line given* DG.

Rom the point A, draw AE parall. [d] to DG, join the points AD. From the point G, draw E parall. [d] to AD. The line AE is = DG, cauſe in the Pgr. [e] DE, the oppoſite ſides [f] E = DG. *Q. E. F.*

[d] *Preced.*
[e] *Def.* 20. *by Conſtr.*
[f] *XXX.*

Problem III.

rom a line given, [a b] *to cut off a part* [a f] *equal to another given* [c d].

Rom the point A, draw AE = [g] CD, Cent. A. interval CD, (or AE) deſcribe the Arch F. The line FA is = CD, becauſe FA = [h] E = [i]) CD. *Q. E. D.*

[g] *Prec.*
[h] *Radius's.*
[i] *Conſtr.*

Problem

Problem IV.

To make an Ang. [g] = *to an An given* [b].

From the line L F, take the part * L D = A B and D E = A C and E F = C B, th on the Cent. $\left\{\begin{matrix} D \\ E \end{matrix}\right.$ interv. $\left\{\begin{matrix} D L \\ E F \end{matrix}\right.$ describe Arch cutting in G, joyn G D, G E. The Ang. G = B, (because the △ D G E is equal [m] sided A B C.) *Q. E. F.*

* *Prec.*

[l] *XVI.*

[m] *Constr. & Ax. 6.*

Problem V.

To divide an Ang. given, [b] *in two equ parts.*

Cent. B, at any Interv. describe the Arch A Cent's. A and C, same Interv. desc. Arc crossing in D, joyn D A, D C, and draw D which divides the Ang. D in two = [n] pa (For the △s B A D, B C D are equal sided, sides B A D, B C D, being Radius's of = and D B common, therefore the Ang. C D = A D B.) *Q. E. F.*

[n] *XVI.*

Probl

Problem VI.

From a point given, [a *or* d] *to raiſe, or let fall a Perp.* [a d].

I. FRom each ſide of D take equals,[o] DB, DC. Cent. and interv. B, C, deſcribe Arches croſſing at A, joyn A D, which is the Perp. required. For A B is = [p] A C, D B = [q] D C, D A common. Therefore the Angles at D are = [r], by conſeq. ∟ [s] and D A is perp.[t]

II. Cent. A, at any interv. deſcr. the Arch A C, joyn A B, A C, divide the ∠ A in the [u] middle, by the time A D which is the perp.[t] required. For the Angles at D, may be proved =, as before. Therefore, &c. *Q. E. F.*

[o] *Prob. III.*

[p] *Radius's*
[q] *Conſtr.*
[r] *XVI.*
[s] *I.*
[t] *Def. 7.*
[u] *Pr. V.*

Problem VII.

To divide a line [a b] *in the middle.*

CEnt. A B, any interv. deſcribe Arches croſſing in D, let fall the perp. D C[w]. This divides A B in the middle. For A D = [x] B D. D C, com. the Angles at D are = [y]. Therefore the ſide C A is = [z] C B. *Q. E. F.*

[w] *Prec.*
[x] *Radius's.*
[y] *Def. 7.*
[z] *XXIV.*

Problem

Problem VIII.

To divide a line [a b] *in a given Proportion.* $\frac{CA}{EA}$

A B, A C, being placed at any Ang. A, joyn B C, and from the point E, draw a Parall. [a] to it, *viz.* E D. Therefore [b] B A . D A : : C A . E A. *Q. E. F.*

[a] *Pr. I.*
[b] *LXVII.*

Problem IX.

To find a fourth proportional, [d e] *to three given,* viz. A B . B C : : A D . d e

Joyn B D, and from the point C, draw its Parall. [c] C E, joyn D E, the fourth proport. for [d] A D . B C : : A D . D E. *Q. E. F.*

[c] *Pr. I.*
[d] *LXVII. divided.*

Problem X.

To find a third Proport. [d e] *to two given,* [a b . b c]

Take A D = [e] B C, and joyn it to the point A, at any Ang. draw D B, and its Parall. [f] C E, from the point C. D E is the third Proport. for [g] A B . B C : : (B C, *i. e.*) A D . D E. *Q. E. F.*

[e] *Pr. II.*
[f] *Pr. I.*
[g] *LXVII divided.*

Problem

Problem XI.

To find a middle proportion [b c] *between two given* [a b b c.]

FRom the point B raiſe the perp. [h] B E. divide A C in the [i] middle, D. Cent. D, interv. deſcribe a Semicirc. cutting the perp. in E. E B will be the middle proport. For (E A, E C, being joyn'd) the ang. A E C is [k] ∟ therefore [l] the perp. E B is a mid. propor. between the parts of the baſe A B C. *Q. E. F.*

h *Pr. VI.*
i *Pr. VII.*
k *LXI.*
l *LXXVI.*

Problem XII.

To divide a line given [Z] *in extreme and middle reaſon. So that* Z, A, E ∴ *and* Z × E = Aq.

FRom C, raiſe the perp. [m] C B = [n] Z. divide C B in the mid. in D. take [o] D F = [n] D H; and C G = C F. G, is the point of diviſion.

m *P. VI.*
n *P. III.*
o *Dem. II.*

For, □ B C F + C Fq + D C q [p] = p *CXI.*

(D F q
D F q *i.e.*[q] D Hq[r] =) Z q + D C q q *Conſtr.*

therefore (omitting the com. D C q) remains r *CXIII.*

B C F + C F q *i.e.*[q] Z A + Aq = Z q

But Z q [f] = Z A + Z E; therefore (Z A being com.) Z E is = [t] A q, and by conſequence [u] Z, A, E, are ∴. *Q. E. F.*

f *CIV.*
t *Ax.* 8.
u *LXXXI.*

Problem XIII.

1. *To make an equilateral triang.* [a b c.]
2. *To make a triang. of lines given.* [d a b e.]

1. DRaw A B. Cent. A and B describe arches crossing in C. joyn C A, C B. therefore [w] all the sides are = *Q. E. F.*

[w] *Radius's of* = ○s.

2. Cent. A and B; interv. A D, B C, describe arches crossing in C. joyn C A, C B. therefore [w] A C = A D, and C B = B E. *Q. E. F.*

Problem XIV.

To make a Pgr. ([g h] *at an ang. given* [d]) = *to a* △ *given* [a b c.]

DRaw B H. [x] pall. A C. divide A C in the mid. [y] G. and make the ang. C G F = [z] D. draw C H pall. G F. and the diag. G H. (The △ G H C [a] =) ½ pgr. F C is [b] = ½ the △ A B C. therefore the whole pgr. F C is [c] = whole △ A B C. *Q. E. F.*

[x] *P. I.*
[y] *P. VII.*
[z] *P. IV.*
[a] *XXX.*
[b] *LXXVIII*
[c] *LXIII.*

Problem

Problem XV.

To make a Pgr. [x] (*on a ſide,* [a b] *and ang.* [c] *given*) = *to a* △ *given* [D.]

MAke S = [d] D, the ang. [b] being *d* = *c.* on the line *b a*, fill [e] up the Pgr. *f a.* Run *e h* through *b*, and continue [e] to it, *f g.* from *h*, draw *h l*, pall. [f] *g a.* and through *b*, draw *i k*, [f] pall *f h*, and continue *e a* to *l.* X is the pgr. requir'd. For, the △ O X R = [g] Z S V. but O = [g] Z. R = [g] V. Therefore the remains [h] X = (S [i] =) D. and the ang. *a b k* is = [k] (*b* [i] =) *c.* *Q. E. F.*

d *Prec.*
e *Dem. II.*
f *P. I.*
g *XXX.*
h *Ax.* 8.
i *Conſtr.*
k *III.*

Problem XVI.

To make a Pgr. [e f g] (*on a ſide* [hi] *and ang.* [d] *given*) = *to a rectilinear figure given* [a b c.]

DIvide the rectilinear figure into △s, A, B, C; Then [k] make the pgr. E = A, according to the ang. D. and the ſide H I. Alſo F = C, and G = D: Theſe 3 will fall between the ſame parallels, becauſe [l] their angles are =, and by conſeq. make one Pgr. = [m] A B C.

k *Prec.*
l *VI.*
m *Ax.*

Problem XVII.

To frame a Square on a line given [a b.]

FRom A raiſe a perp. [n] A C = [o] A B. Cent. B and C interv. B A. deſcr. arches croſſing at D, joyn D C, D B. *Q. E. F.* For, A B = [q] A C = [r] C D = [r] D B. C D is com. therefore [f] the △s A C B, C B D, are equiangled. Therefore A [q] = △ (L = [f]) D. but ∠ A C B is = [t] A B C, and D C B = [r] D B C. each [u] = ½ a ∟s. therefore C and B are ∟, and the oppoſite ſides are pall [w]. *Q. E. D.*

[n] *P. VI.*
[o] *P. III.*
[q] *Conſtr.*
[r] *Radius's.*
[f] *XVI.*
[t] *XIII.*
[u] *X.*
[w] *VI.*

Problem XVIII.

To make a □ [d g, q] = *to a right lin'd figure given.*

MAke [x] the □ B C = A. Between the ſides of this □ find a mid. propor. [y] D G. The ſquare of this is = [z] (B C, =) A. by conſtr. *Q. E. F.*

[x] *P. XIV.*
[y] *P. XI.*
[z] *CXII.*
Schol.

Problem

Problem XIX.

To make a □ [ed], = *to a right-lin'd-figure* [a], *and* □ *given* [bc]; *both together.*

MAke the □ EF = [a] A, and place the corner of it, G, againſt the □ BC, ſo that the ∠ EGB, may [b] be ∟, on the Subtend, EF, [a] make the □ ED, (which will be) [c] = (EF [d] =) A + BC. *Q. E. F.*

[a] *Prec.*
[b] *III.*
[c] *CXIII.*
[d] *Conſt.*

I. *Hence appears how one Square may be ſubtracted from another.*

II. *How a great many Squares given, may be reduced to one Square. For the* □ F, *is* [c] = Eq. (+ Iq. =) Dq. (+ Kq. =) Cq. (+ Lq. =) Bq. + Aq.

Problem XX.

To frame a Pentagon upon a line given [A B.]

DIvide A B in the middle, C; raiſe the perp. C D = A B. Draw out D A to I, ſo that A I may be = A C. upon the baſe A B, make the △ A B F each ſide = D I. Laſtly, upon theſe ſides make the △s F G B, A H F, each ſide = A B. A B G F H is the Pentagon required. For,

Take F K = A B. and on the cent. A. interv. A C, deſcribe the ◯ I C L. The line D I is [a] = F A, and if you take away the [a] = 's F K, I L, there [b] remains K A = L D. Now D C is a *tangent* [c] to the ◯. Therefore [d], I D. D C :: D C. D L. that is, F A. A B :: A B. A K. Wherefore [e] the △ A B F is :: A B K. by conſeq. [f] the ∠ ζ = β. and κ = (ζ × n [g] =) ε. Therefore the ſide B K = (B A =) K F. Therefore the ∠ β = [h] n. *viz.* ζ = β = n. But the external ∠ κ [i] = β × n, therefore κ, (or ε, or ζ × n) are each double to β. But ε, ζ × n, β, together, are [k] = 2 ∟, therefore β = ⅓ 2 ∟ and κ = ⅔ 2 ∟ (being double to β) therefore λ = [l] 1⅓ 2 ∟. but λ [m] = G [m] = H.

Again, the ∠s F G B, F K B, F H A, are [l] =. (the ſide B K being prov'd = B A) therefore the ∠s θ, n, δ, α, γ, and β, are all = to one another, and by conſeq. = ⅕ 2 ∟. Therefore δ ε, ζ n θ, α β γ, are each = ⅗ 2 ∟

a. By *conſtr.*
b *Ax.* 8.
c *XLIX*, *conſtr.*
d *CXXII.*
e *LXXIV.*
f *Def.* 45.
g *XII.*
h *XIII.*
i *IX.*
k *X.*
l *I.*
m *XVI.*

2 ∟. as H x G have already been prov'd. So that all the 5 ang.'s, are [n] =. And the 5 sides are = by construction. Therefore, *Q. E. F.*

[n] *Ax.* 10.

Scholium.

The line F A *is cut in extreme and middle reason in the point* K; *for* F A. (A B, i. e.) F K :: F K. K A. *as appears in the precedent.*

Problem XXI.

To make a regular Hexagon, on a line given [a b.]

ON A B, [e] make an equal sided △ A B C. as also on A C, and B C. then drawing [f] out A C and B C, make C F and C E [g] = to them, and joyn G F E D. *Q. E. F.* For, the 3 ang.'s H C I, are = [h] 2 ∟. therefore G D is a [i] right line. Further, the ang. L is = [k] C, and L F, L E are = [l] C A, C B, therefore the △ L F E is = [m] C A B, and so of the other △s, and by conseq. the two angles at E, shall be = 2 at A, and F E = A B, *&c.* So that the whole figure is equal sided and equiangled. *Q. E. D.*

[e] *P. XIII.*
[f] *Dem. II.*
[g] *P. III.*
[h] *XVI, with X.*
[i] *II.*
[k] *III.*
[l] *Constr.*
[m] *XVII.*

Problem XXII.

To make a Polygon (on a line given) [a b] like, *and alike placed, to a Polygon given* [g d.]

DIvide DG into △s, and on AB make the ∠ ABH [n] = CDE; and BAH [n] = DCE, therefore [o] the ∠ H is = E, and by conseq. the △ AHB is :: [p] CED, and so of the rest: Therefore [q] the whole DK is :: GD. *Q. E. F.*

[n] *P. IV.*
[o] *XVII.*
[p] *LXXI.*
[q] *LXIII.*

Problem XXIII.

To make a Polygon [c] like, *and alike placed, to a Polyg. given* [A], *and = to another Polyg. given* [B.]

ON the line DE, make [r] a Pgr. [f] = A, and on EL, the pgr. Z = [f] B. Take HI a mid. [t] propor. between DE, EK, and make on it the Polyg. C [u] :: A. This C is = B. For, $\frac{A}{C} = {}^{w}\left(\frac{DE}{EK}\ twice.\ \text{i.e.}^{x}\ \frac{DE}{EK} = {}^{w}\frac{S}{Z}\ \text{i.e.}^{y}\right)\frac{A}{B}$ Therefore C [z] = B. *Q. E. D.*

[r] *P. XVI.*
[f] *P. XIV.*
[t] *P. XI.*
[u] *Preced.*
[w] *XCI.*
[x] *Constr. bec. DE, HI, EK, are* ∴.
[y] *Ax.* 10. *constr.*
[z] *Ax.* 10.

Problem

Problem XXIV.

To find the center of an Arch (or Circle) given [a b c] (*or, to draw a* ○ *through 3 points given.*)

TAke 3 points in the arch. *viz.* A, B, C. joyn A B, B C. divide these in [c] the mid. [a] D and F, by [b] perp's, crossing in E, which is the Center. For each perp. is a [c] Diam. and by [d] conseq. the Center must be where they cross.

[a] *P. VII.*
[b] *P. VI.*
[c] *XXXVII.*
[d] *Def.* 22.

Problem XXV.

To divide an Arch [a c] *in the middle.*

HAving found the Cent. [e] D, draw the Radius A D C, and divide the ∠ D in the [f] mid. by the line D B; therefore the arch A B is [g] = B C. *Q. E. F.*

[e] *Prec.*
[f] *V.*
[g] *XCVI.*

Problem

Problem XXVI.

To draw to Tangent [a d] *from a point given* [a.]

[h] *P. XXIV.*

DRaw a line from A to the Cent. [h] B, on A B; descr. a Semicirc. cutting the ◯ in D; joyn A D, B B. A D, is the Tangent. For the ∠ A D B is ∟ [i]; therefore [k] *&c. Q. E. F.*

[i] *LVI.*
[k] *XLIX.*

Problem XXVII.

To cut off a Segment [a b] *from a* ◯ *given, that may receive an* ∠ = *to an ang. given,* viz. A C B = E.

[l] *Prec.*
[m] *P. IV.*
[n] *LX.*
[o] *Const.*

DRaw a [l] Tangent F G, touching in B, make the ∠ F B A = [m] E. Therefore [n] the ∠ A C B (in the opposite Segment) is = (F B A. [o] =) E. So that A D C B is the Segment required.

Problem

Problem XXVIII.

To make a △ *in a* ○, :: *a* △ *given* [def.]

CUt[p] off, by AB a Segment receiving an ang. ACB = to the ∠ given, F: Then make BAC[q] = D, joyn BC, the ∠ BCA is = [r] F. Therefore E[f] = B, and by conseq. △ABC :: [t] DEH *Q. E. F.*

[p] *Prec.*
[q] *P. IV.*
[r] *LIV.*
[f] *XXII.*
[t] *LXXI.*

Problem XXIX.

To make a △ *about a* ○ = *a* △ *given* [def.]

MAke the ∠ at the cent. BGA[u] = EDH, and CGA[u] = EFI; and at the ends of the 3 Radius's A, B, C, draw[w] perps, meeting in K, L, N. *Q. E. F.* For, in the *Trapeze*, KBGA, the 4 ∠s are[x] = 4 ∟, (being by a diagonal divisible into 2 △s.) but B and A are[y] ∟; therefore K, G = 2 ∟. Now G = [y] EDH, therefore K = [z] EDF. So also M is = EFD, therefore[a] L = E; and by conseq. [b] the △s are like. *Q. E. F.*

[u] *P. IV.*
[w] *P. VI.*
[x] *X.*
[y] *Constr.*
[z] *I.*
[a] *XXII.*
[b] *LXXI.*

Problem

Problem XXX.

To inſcribe a ○ in a △ given [a b c.]

[c] *P. V.* DIvide the ∠s A and C in the mid.[c] by lines croſſing in D. From D, draw perps. to the ſides of the △, or D E, D F, D G; cent. D, interv. D F, deſcr. the ○. *Q. E. F.* For, In the △s A E D, A D F, the ∠s at A are [d] *Conſtr.* [d] =: Alſo A F D, A E D are [e] =, being [e] *Ax. 2.* ∟ [d], and the ſide A D is com. therefore [f] the [f] *XXIII.* ſide D E is = D F. = (for the ſame reaſon) D G. But D F is a Radius [d], therefore alſo [g] *XLVII.* muſt be D E and D G: and by conſeq. [g] the ſides B A, B C touch the ○, as A C does, by conſtr. *Q. E. F.*

Problem XXXI.

To conſcribe a ○ about a △.

DRaw a Circle through 3 points, by P. XXIV.

Problem

Problem XXXII.

To inſcribe a □ *in a* ⊙.

DIvide the Diam. A C in the middle[h], by the perp. B D[i]; joyn B, C, D, A, which is, the □ ſought. For the 4 ſides are[k] =, ſubtending[l] = ∠s (at the Cent.) between = ſides[m], and the 4 ∠s A B C D are ∟[n]; for A C is a Diam[o]. *Q. E. D.*

h *P. VII.*
i *P. VI.*
k *XVII.*
l *Ax. 2.*
m *Radius's.*
n *LVI.*
o *XXXVII*

Problem XXXIII.

To conſcribe a ⊙ *about a given* □.

DRaw Diams. croſſing perpendicularly[p] at the Cent. I, and at their ends, E, F, G, H, draw 4[q] perps. meeting in A, B, C, D, which is the □ ſought. For, ∠ B E G[q] = E G D; therefore B A is[r] pall. C D, in like manner B C is pall. A D, and A D pall. E G, and by conſeq. the ∠ B A H is (=[f] B E I) ∟[t], and the ſide A D[u] = E G = F H[u] = B A, *&c.* thus all the ſides will be prov'd =, and all the angs. ∟. *Q. E. D.*

p *P. VII, and VI.*
q *P. VI.*
Ax. 2. con.
r *VI.*
f *IV.*
t *Conſtr.*
u *XXX.*
w *Diams.*

Problem

Problem XXXIV.

To inſcribe a ○ in a given □.

x *P. VII. and VI.* DIvide the ſides AB, BC, in the x mid. by by perps. croſſing at I. Cent. I, interv. IE, deſc. a ○. *Q. E. F.* For, ∠ BEG is y ∟ z, = EGD; thus the ∠ FHD, is ∟ alſo: Therefore IH is (pall. a, and = b EA, = y EB) = a b FI = (by the ſame proceſs) IG = IE. But IE is a Radius y, therefore alſo muſt all the reſt be, (being = to IE) by conſeq. the ○ touches all the ſides of the □. *Q. E. D.*

y *Conſtr.*

z *IV.*

a *VI.*

b *XXX.*

Problem XXXV.

To conſcribe a ○ about a regular Pentagon.

c *P. V.* DIvide the ∠s A and B in the mid. c by lines croſſing in F. Cent. F, interv. FB, deſcr. a ○. *Q. E. F.* For, the ∠ FAB is = d FBA. Therefore FB = e FA, = h FC; becauſe in the △s HG, the ſide AB = f BC, BF com. ∠ FBA g = FBC. Alſo, FB is h = FD. For, in the △s GI, BC f = BD; FC, com. ∠ FCB = FCD. (For FB is proved = FC; therefore ∠ FBC = i

d *Ax. 7.*

Conſtr.

e *XII.*

f *Suppoſ.*

g *Conſtr.*

h *XVII.*

= [i] FCB; therefore FCB is ½ an ang. of the Pent. and by conſeq. ſo is FCD.) Thus all the lines FC, FD, &c. will be proved = FB, which is a Radius [g], therefore alſo are they; and conſeq. the ○ paſſes by all the angs. of the Pentag. *Q. E. D.*

i *XIII.*

Problem XXXVI.

To inſcribe a ○ *in a Pentagon.*

CEnt. F. [k] interv. FM (*viz.* a perp. [l] from the cent. to the ſide) deſcr. a ○. *Q. E. F.* For, the ∠s at E are [k] =; and (perps. being [l] drawn to all the ſides) to ∠ FME = FNE, and the ſide FE, com. [m] therefore FM is = FM: thus all the perps. will be proved = to FM, which is a Radius [n]; therefore alſo are they all: and by conſequence the ○ touches all the ſides of the Polygon. *Q. E. D.*

k *Preced.*
l *P. VI.*
m *XXIV.*
n *Conſtr.*

Problem XXXVII.

To inſcribe a (regular) Pentagon in a ○ *given.*

MAke a reg. Pentag. [o] EB, whoſe cent. F being found, lay it upon the cent. of the ○; draw the Radius's FH, and through the angs.

o *P. XX.*
p *P. XXXV*

angs. of the Pentag. joyn the points G, L, K, &c. *Q. E. F.* For, [q] $\frac{GF}{HF}$ (Radius)^q = $\frac{EF}{DF}$ (bec. EF [r] = DF.) therefore GH is [s] pall. to ED, and by conseq. [t] $\frac{HF}{DF} = \frac{GH}{ED}$ = (by the same reason) $\frac{HI}{DC}$ &c. But ED = DC, therefore GH = HI, &c. Lastly, upon account of the pall. lines, all the angs. are [u] = respectively, and (by consequence) between themselves; becaufe all the ∠s in the given Pentagon are [w] =. *Q. E. D.*

[q] *Ax.* 10.
[r] *Prov'd in P. XXXV.*
[s] *LXXVIII*
[t] *LXIX.*
[u] *IV.*
[w] *Suppos.*

Problem XXVIII.

To inscribe a Hexagon in a ○.

AT the interval of the Radius FA, cut off the arch AB, draw the Rad. FB, therefore [s] is an equal [x] sided △. Then at the interv. of the Rad. FB cut off the arch B, C, &c. therefore Z is equilat. △, in like manner X. &c. therefore DA is a right [y] line, and by conseq. a Diam. But the other half ○, DEA, will admit 3 △s = to the former; and by conseq. the whole ○ admits a regular [z] Hexagon. *Q. E. D.*

[x] *Constr. Rad.*
[y] *X, & II.*
[z] *XXI.*

Problem

Problem XXXIX.

A Polygon in a ○ [b c d] *being given, to inſcribe a* like *Polygon* [e f g] *in a* ○ *of any different ſize.*

UPon the center of the given ○, A, deſcribe the other ○, E F G; draw the Radius's through the ∠s of the Polyg. given, B, C, D, *&c.* joyn the ends of theſe Radius's with the right lines E F, F G, *&c.* The Polyg. E F G will be like the Polyg. B C D. For the △ A B C is :: [o] A E F.

Becauſe the 3 ∠s at A, B, and C, together, are [p] = (2 ∟ [p] =) A, E and F together; therefore taking away from theſe |= ſumms, the common B, there remains B C = [q] E F. But, B = [r] C, and E [r] = F, therefore B and E, (being like parts, *i. e.* halfs, of = ſumms) are = [f]; therefore the line E F is [t] pa ll. to B C.

In like manner, all the other △'s will be proved *like*; therefore the whole [u] Polygons are like. *Q. E. F.*

[o] *LXX.*
[p] *X.*
[q] *Ax.* 10.
[r] *XIII.*
[f] *LXIII.*
[t] *VI.*
[u] *LXVII.*

End of the Problems.

To be Corrected and Added.

IN the Advertiſement, l. 3. r. *Principles of:* Def. 6. l. 8. r. *Angle S:* l. 9. after *are equal*, add, 3. *The meaſure of this inclination is (either an arch of a Circle*, BC, *of which the ang.* [a] *is the Cent. or elſe) a ſtrait line* [gh]; *which being lengthned or ſhortned (the points* g, h, *remaining ſtill at the ſame diſtances from the ang.* a) *will make the inclination of the lines*, ab, ac, *more or leſs than it is; but if the line* gh, *remains the ſame, and in the ſame place, the inclination will be the ſame. And by conſeq. if* IK *be of the ſame length with* GH, *and the points* IK *at the ſame diſtances from the ang.* D; *as* GH *are reſpectively from* a; *then* DI, DK, *have the ſame inclination with* AG, AH. Def. 30. marg. add *Fig. XVI:* l. 2. r. [ac, bc.] Def. 31. marg. dele *Fig. XVI.* Def. 46. at the end, add, *And not both Antecedents, and both Conſequents in either one of the figures.* Ax. 3. marg. add *Fig. VI.* at the end, r. [bc]. Explication of the notes, at the end,

Q. E. D.

Q. E. D. which was to be demonſtrated. Q. E. F. *which was to be done.* Pa. 1. l. laſt but 4, r. *Perp.* p. 2. l. 9. r. ABE. p. 8. l. laſt but 5. r. BC = EF: p. 15. to Theor. XXIX. add, *Note, the deſign of this and the* XXVII. *is not to ſhew what lines are pall. but to prove that* = △*s, upon* = *baſes, are of the ſame height.* (Def. 47.) *becauſe all perps. between the ſame palls. are* =, *for ſuch perps. making* (∟, *that is*) = ∠*s, will be pall. themſelves* (VI.) *and by conſeq.* = (XXX.) p. 72. l. laſt but 4. for $\frac{CD}{FI}$ r. $\frac{CD}{HI}$.

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